Searching singlet extensions of the supersymmetric standard model in Z_{6-II} orbifold compactification of heterotic string[*]

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We search for supersymmetric standard model realizations with extra singlets and extra $U(1)$ using the heterotic string compactification on the Z_{6-II} orbifold with two Wilson lines. The effective superpotential produced through the vacuum restabilization mechanism is examined for three representative Pati-Salam string models obtained in the literature. An automated selection of semi-realistic vacua along flat directions in the non-Abelian singlet modes field space is performed by requiring the presence of massless pairs of electroweak Higgs bosons having trilinear superpotential couplings with massless singlet modes and the decoupling of color triplet exotic modes needed to suppress B and L number violating processes.

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I. INTRODUCTION

Model building with the heterotic string [1–9] has received a vigorous stimulus thanks to the recent focus [10–26] on anisotropic compactifications on the orbifold Z_{6-II}. Recent reviews are given in [27, 28]. Satisfactory representative vacuum solutions are also reported for the Z_{12-I} orbifold [29–33]. The exploration of the vast moduli space of the Z_{6-II} orbifold with two authorized Wilson lines [17, 20] was found to give access to a fertile mini-landscape of vacua for minimal supersymmetric standard models descending from 5-d or 6-d grand unified theories. Sampling the still wider space of solutions with three Wilson lines [34] leads to equally hopeful conclusions.

One characteristic feature of these top-down constructions is the profusion of $U(1)$ gauge factors which remain after the first gauge symmetry breaking from the orbifold gauge twist and the presence of Wilson lines. One of them is anomalous, creating a one-loop Fayet-Iliopoulos D-term a bit below the string scale that needs to be cancelled. This step usually leaves a lot of freedom due to the large vacuum degeneracy. In the vacuum restabilisation process, the $U(1)$ gauge symmetries will get broken and most of the extra modes beyond the MSSM (minimal supersymmetric standard model) matter content, but hopefully not all of them, will decouple with large masses. Except for searches of extensions including right-handed neutrino supermultiplets [35, 36], most studies have restricted to solutions in which the whole set of Standard Model singlet modes acquire large masses and decouple from the low energy field theory. It is natural, however, to ask whether the large moduli space of these compactifications does not include regions where extra singlets and extra $U(1)$ symmetries might realize the NMSSM (next-to-minimal supersymmetric standard model) or one of the related versions proposed in the literature [37–42] and analyzed over the years in phenomenological [43–48] and formal [49–53] studies. (After completion of the present work, there appeared a study of NMSSM realizations complying with the severe selection criteria of [17, 20] which reports that solutions exist only in the case with three Wilson lines [54].)

Filling this gap is the purpose of the present work. Rather than pursuing a statistical exploration of the moduli space of heterotic string compactifications on the Z_{6-II} orbifold, we restrict consideration in this preliminary study

to the three representative 5-d Pati-Salam models, designated in [10] as models $A1$, $A2$, B, and apply techniques developed in past works. The early studies along these lines focused on flat directions for the non-Abelian singlet modes realizing the minimal supersymmetric standard model [55–58] or the Pati-Salam model [59], while subsequent studies have explored field directions along scalar modes charged under the non-Abelian gauge group factors [60–63].

Having in hand the string massless spectra for a set of representative solutions, we use a Fortran computer program to implement an automated search of the flat directions which exhibit a satisfactory supersymmetric effective action. Our study is somewhat close in spirit to that of [59]. Concentrating on string theory realizations [64–67] of the Pati-Salam model [68] comes with a number of well-known advantages. We shall not detail in the present work the Pati-Salam to Standard Model ($PS \to SM$) gauge symmetry breaking, but note that the use of Pati-Salam group multiplets merely serves a convenient book-keeping purpose, since the representation content with respect to the Standard Model gauge group is then uniquely determined.

The contents of the present work are organized into four sections. Section II discusses general features of the low energy theory with Pati-Salam gauge symmetry. A brief review of the supersymmetry preserving flat directions in the presence of an anomalous $U(1)_A$ gauge symmetry is provided in Appendix A. In order to clarify certain fine points that have caused us much confusion, we found useful to include in Appendix B a brief review of the string theory construction for the Z_{6-II} orbifold complementing the detailed discussions in [10, 13, 20]. The core part of the present work figures in Section III, where we present the results of our searches of singlet extended supersymmetric standard models for the Pati-Salam string models $A1$, $A2$, B of [10]. A general discussion of results and a summary of our conclusions figures in Section IV.

II.   LOW ENERGY THEORY

A.   General features of Pati-Salam string models

Our interest is on the 4-d orbifold compactifications of the $E_8 \times E_8'$ heterotic string satisfying $\mathcal{N} = 1$ supersymmetry, with gauge symmetry group, $G \times G'$, including a Pati-Salam observable sector,

$$G = G_{422} \times \prod_{a=1}^{N_a} U(1)_a, \qquad [G_{422} = SU(4) \times SU(2)_L \times SU(2)_R] \tag{II.1}$$

and a hidden sector,

$$G' = SO(10)' \times SU(2)' \quad \text{or} \quad G' = SO(10)', \tag{II.2}$$

with the corresponding Abelian factors, $U(1)^{N_a}$, $[N_a = 5,6]$. The gauge group representations consist of three chiral generations of bifundamental matter supermultiplets, $f_i \sim (4, 2, 1)$, $f_i^c \sim (\bar{4}, 1, 2)$, a number of electroweak Higgs bidoublet supermultiplets, $h_i \sim (1, 2, 2)$, and a number of vector bifundamental supermultiplets, f_i', $\bar{f}_i'$ and $f_i'^c$, $\bar{f}_i'^c$, which accomplish the $PS \to SM$ Higgs mechanism for the gauge symmetry breaking, $SU(4) \times SU(2)_R \to SU(3)_c \times U(1)_Y$, along with a set of non-Abelian singlet modes, $\phi_i \sim (1, 1, 1)(1', 1')$. There also occur modes with exotic quantum numbers:

$$C_i \sim (6, 1, 1), \quad d_i^r \sim (1, 1, 2), \quad d_i^l \sim (1, 2, 1), \quad q_i \sim (4, 1, 1), \quad \bar{q}_i \sim (\bar{4}, 1, 1); \tag{II.3}$$

hidden sector modes:

$$\Sigma_i' \sim (16', 1'), \quad \bar{\Sigma}_i' \sim (\overline{16'}, 1'), \quad T_i' \sim (10', 1'), \quad d_i' \sim (1', 2'); \tag{II.4}$$

along with modes charged under the observable and hidden groups:

$$h_i'^l \sim (1, 2, 1)(1', 2'), \quad h_i'^r \sim (1, 1, 2)(1', 2'). \tag{II.5}$$

Among the set of non-Abelian singlets, ϕ_i, we distinguish the modes without and with oscillator excitations, S_i and Y_i. We group the singlets into two subsets, the first including the fields $\hat{\phi}_i$ excited along the flat directions with large vacuum expectation values (VEVs) of the order of the string or Planck mass scales, m_s or $M_\star$, and the second including the dynamical fields, $\tilde{\phi}_i$ and ϕ_i, with undetermined or vanishing VEVs and masses.

The $PS \to SM$ phase transition is assumed to occur at a mass scale m_{PS} near the string scale. This hypothesis is in harmony with the renormalization group analysis of the gauge coupling constants [10] and with related phenomenological studies [65–67]. At the $PS \to SM$ gauge symmetry breaking, the above listed multiplets decompose into Standard Model group $G_{321} = SU(3) \times SU(2)_L \times U(1)_Y$ multiplets as,

$$f \sim (4, 2, 1) \to q + l = (u\ d) + (\nu\ e) \sim (3, 2)_{\frac{1}{6}} + (1, 2)_{-\frac{1}{2}},$$

$$
\begin{aligned}
f^c &\sim (\bar{4},1,2) \rightarrow q^c + l^c = (d^c\ u^c) + (e^c\ \nu^c) \sim (\bar{3},1)_{\frac{2}{6}} + (\bar{3},1)_{-\frac{4}{6}} + (1,1)_1 + (1,1)_0, \\
h &\sim (1,2,2) \rightarrow H_u + H_d \sim (1,2)_{\frac{1}{2}} + (1,2)_{-\frac{1}{2}}, \\
C &\sim (6,1,1) \rightarrow \bar{g}^c + g^c \sim (3,1)_{-\frac{1}{3}} + (\bar{3},1)_{\frac{1}{3}}, \\
q &\sim (4,1,1) \rightarrow g' + E = (3,1)_{\frac{1}{6}} + (1,1)_{-\frac{1}{2}}, \\
d^l &\sim (1,2,1) \rightarrow (N^l\ E^l) \sim (1,2)_0, \\
d^r &\sim (1,1,2) \rightarrow E^r + N^r \sim (1,1)_{-\frac{1}{2}} + (1,1)_{\frac{1}{2}}.
\end{aligned}
\tag{II.6}
$$

Analogous decompositions hold for the conjugate representations, $\bar{f} \sim (\bar{4},2,1)$, $\bar{f}^c \sim (4,1,2)$, $\bar{q} \sim (\bar{4},1,1)$. The superpotential couplings of Pati-Salam modes of lowest order decompose into couplings between SM modes as

$$
\begin{aligned}
h_k h_l &= H_{u,k} H_{d,l} + H_{d,k} H_{u,l}, \\
q_k \bar{q}_l &= g'_k \bar{g}'_l + E_k \bar{E}_l, \\
C_k C_l &= \bar{g}^c_k g^c_l + g^c_k \bar{g}^c_l, \\
h f f^c &= H_u q u^c + H_d q d^c + H_d l e^c, \\
C f f &= \bar{g}^c q q, \\
C f^c f^c &= g^c u^c d^c + \bar{g}^c u^c e^c + \bar{g}^c d^c \nu^c, \\
C \bar{f}^c \bar{f}^c &= \bar{g}^c \bar{u}^c \bar{d}^c + g^c \bar{u}^c \bar{e}^c + g^c \bar{d}^c \bar{\nu}^c.
\end{aligned}
\tag{II.7}
$$

B. Search strategy

We search vacua of the three chiral generation models $A1$, $A2$, B of [10] with the following characteristics at low energies:

- One or several non-Abelian gauge group singlets, ϕ_k, coupled to one or several pairs of electroweak Higgs boson bidoublets by trilinear effective superpotential terms, $h_i h_j \phi_k$;

- Extra $U(1)'$ symmetries [48] possibly broken at mass scales much lower than $O(M_\star)$;

- A secluded sector with additional singlets [39, 42] belonging to an approximate flat direction and charged under the extra $U(1)'$ only. The simplest version includes in addition to s three singlets [39, 42] s_1, s_2, s_3 belonging to a flat direction approximately lifted by a small coupling constant λ' in the effective renormalizable superpotential, $W_{EFF} = \lambda H_u H_d s + \lambda' s_1 s_2 s_3$.

The superpotential is decomposed into three main components, W_s, W_2, W_3, associated to the couplings of the non-Abelian singlet fields, ϕ_i, alone or multiplied by two and three fields charged under the Pati-Salam group. Each component is built as a power expansion in the ϕ_i of the form

$$
\begin{aligned}
W_s(\phi_k) &= \sum_n W_s^{(n)}(\phi_k) = \sum_{n,m} s_m^{(n)} \prod_{i,j} S_i^{p_i^{(m)}} Y_j^{q_j^{(m)}}, \\
W_2(\phi_k) &= r_{ij}(\phi_k) f_i^c \bar{f}_j^c + p_{ij}(\phi_k) f_i \bar{f}_j + \mu_{ij}(\phi_k) h_i h_j + \tau_{ij}(\phi_k) C_i C_j + \sigma_{ij} q_i \bar{q}_j, \\
W_3(\phi_k) &= C_k [c_k^{ij}(\phi_l) f_i^c f_j^c + c_k^{'ij}(\phi_l) \bar{f}_i^c \bar{f}_j^c + l_k^{ij}(\phi_l) f_i f_j + e_k^{ij}(\phi_l) q_i q_j + e_k^{'ij}(\phi_l) \bar{q}_i \bar{q}_j] \\
&\quad + \lambda_k^{ij}(\phi_l) h_k f_i f_j^c + \lambda_k^{'ij}(\phi_l) h_k \bar{f}_i \bar{f}_j^c,
\end{aligned}
\tag{II.8}
$$

where $p_i^{(m)}$, $q_j^{(m)} \in Z_+$, $\sum_{i,j}(p_i^{(m)} + q_j^{(m)}) \geq n$ and the summations over f_i, f_i^c (and their complex conjugates) include both the matter and Higgs multiplets. The coefficient functions $\mu_{ij}(\phi_k)$, $\sigma_{ij}(\phi_k)$, $\tau_{ij}(\phi_k)$, $\cdots$ are given by infinite power expansions in the singlet fields of same form as that displayed above for $W_s^{(n)}$. Integrating out the massive modes C_i, by means of the classical fields equations, can produce baryon and/or lepton number violating couplings represented by local operators of dimension $\mathcal{D} = 4$: lqd^c, $u^c d^c d^c$, lle^c, or of dimension $\mathcal{D} = 5$: $qqql$, $u^c d^c d^c \nu^c$. The resulting dangerous local operators,

$$
\begin{aligned}
W_{EFF} &= (l_k(\tau^{-1})_{kl})\ l_l f f f f + (l_k(\tau^{-1})_{kl} c_l)\ f f f^c f^c + (c_k(\tau^{-1})_{kl} c_l)\ f^c f^c f^c f^c \\
&\rightarrow \frac{1}{M_\star} [l^2(\phi) qqql + l(\phi) c(\phi) qq d^c u^c + c^2(\phi) u^c d^c d^c \nu^c] + \frac{m_{PS}}{M_\star} c^2(\phi) u^c d^c d^c,
\end{aligned}
\tag{II.9}
$$

may compete with similar couplings already present at the compactification scale.

Our search strategy consists of four stages.

- Firstly, we construct (up to some fixed order) the superbasis of holomorphic invariant monomials

$$P_\alpha(\phi) = \prod_i \phi_i^{r_i^\alpha} = \prod_i S_i^{n_i^\alpha} Y_i^{m_i^\alpha}, \quad [r_i^\alpha, \ n_i^\alpha, \ m_i^\alpha \in Z_+] \tag{II.10}$$

  which solve the D flatness conditions, $Q_a(P_\alpha) = 0$, $Q_A(P_\alpha) \times$ sign $(\xi_A) < 0$, where Q_a, $[a = 1, \cdots, N_a - 1]$ denote the anomaly free Abelian charges and Q_A the single anomalous Abelian charge with Fayet-Iliopoulos parameter ξ_A defined in Eqs. (A.1) and (B.9).

- Secondly, we construct the superpotential W_s in the non-Abelian singlet modes (up to some fixed order) satisfying the gauge symmetries and the string selection rules in Eqs. (B.10).

- Thirdly, we scan the D flat monomials $P_\alpha(\phi_i)$ and compare each of these in turn with the allowed couplings of singlets in $W_s(\phi_i)$, in order to determine which of the D flat monomials obey the type A or B F flatness, such that there are no allowed couplings in the superpotential W_s with all field factors, or all but a single field factor, included in $P_\alpha(\phi_i)$. (Said otherwise, type A or B lifting of a flat direction $P_\alpha(\phi_i)$ occurs whenever some superpotential monomial with only 1 or no field having zero expectation value is allowed.) Our terminology slightly deviates from the original one in [55, 56, 62] which referred to the subset of monomials in W_s allowed by the gauge invariance rules alone.

  The conservative approach of exploring the space of supersymmetric vacua consists in restricting to the type B directions, on the grounds that these preserve local supersymmetry, thanks to the vanishing scalar potential or cosmological constant, $< V >= 0$, and can be made flat to all orders by imposing a finite number of conditions. The existing applications [56, 57, 59, 62] typically stop at the trilinear or fourth orders terms of $W_s(\phi)$. An alternative procedure is pursued in [15, 17, 31] which consists in selecting a (typically sizeable) set of singlet field directions providing for a satisfactory effective action and solving next the D and F flatness equations out to high orders for these singlets. In the present work, we follow an intermediate approach in which we restrict to flat directions and superpotential monomials of low orders only but include both type A and B flat directions. Since supersymmetry must get broken anyway, we argue that it makes sense to consider the approximate flat directions which are lifted by non-renormalizable F terms of reasonably high orders. To motivate our choice we note that the condition $< W_s >\approx 0$ is automatically satisfied for models preserving an approximate R symmetry [70]. We also observe that an initially non F flat direction can be repaired into a flat one by using the cancellation with contributions from higher order and/or moduli dependent couplings [71]. One may tie the order of the F term lifting operator to the supersymmetry breaking scale F by considering the case of a single flat direction ϕ lifted by the superpotential, $W_s(\phi) = \phi^{p+3}/M_\star^p$. Assuming that ϕ picks up from radiative corrections a tachyonic soft supersymmetry breaking mass term, $V(\phi) = -m_W^2 |\phi|^2$, then the scalar potential minimization,

$$V(\phi) = -m_W^2 |\phi|^2 + \frac{(p+3)^2}{M_\star^{2p}} |\phi^{p+2}|^2 \implies |F|^{\frac{1}{2}} = V^{1/4}(\phi_{min}) \simeq (m_W^{p+2} M_\star^p)^{\frac{1}{2(p+1)}}, \tag{II.11}$$

  yields the following acceptable range for the breaking scale, $|F|^{\frac{1}{2}} \leq (10^6 - 10^8)$ GeV, for $p = (1 - 3)$. Larger p yield higher scales. It should be noted that the F term lifting by non-renormalizable operators also plays a useful rôle in string theory models realizing gauge symmetry breaking [72, 73] or supersymmetry breaking [74] at intermediate scales.

- Fourthly, we select some supersymmetry preserving flat directions, identified by sets of excited singlet fields $\hat{\phi}_i$, and evaluate the effective superpotential between the non-singlet modes, while treating the fields $\hat{\phi}_i$ VEVs as free parameters. Our main focus will be on the bilinear and trilinear couplings of the electroweak Higgs modes, $h_i h_j (\mu_{ij}(\hat{\phi}) + \lambda_{ij}^k(\hat{\phi})\phi_k)$. However, for completeness, we shall also examine the bilinear couplings of the modes carrying the bifundamental, sextet and quartet representations, in order to test the decoupling of mirror generations and exotic modes and the absence of baryon number violation.

We pause briefly at this point to clarify some technical issues. The non-Abelian singlet modes VEVs are 0-dimensional if they are completely fixed by the gauge constraints, in which case they are roughly set at, $\hat{\phi} = O((-\xi_A/Q_A(\hat{\phi}))^{\frac{1}{2}})$. Otherwise, they may be 1-d or higher if they depend on a single or several free complex parameters.

A given holomorphic monomial of the non-Abelian singlets, $P_\alpha = \prod_{i=1}^n \phi_i^{r_i^\alpha}$, is D-flat if it carries vanishing anomaly free charges, $Q_a(P_\alpha) = \sum_{i=1}^n Q_a(\phi_i)r_i^\alpha = 0$, and a finite anomalous charge, $Q_A(P_\alpha) = \sum_{i=1}^n Q_A(\phi_i)r_i^\alpha \neq 0$ of opposite sign to the charge anomaly, $Trace(Q_A) \propto \xi_A$. A superpotential monomial, $W_x = \prod_{i=1}^n \phi_i^{s_i^x}$, is allowed if it carries

vanishing anomaly free and anomalous charges, $\sum_{i=1}^{n} Q_a(\phi_i)s_i^x = 0$, $\sum_{i=1}^{n} Q_A(\phi_i)s_i^x = 0$, and satisfies the string selection rules listed in Eq. (B.10).

We define the order n of a holomorphic D flat or superpotential monomial, $P_\alpha(\hat{\phi}_i)$ or $W_x(\hat{\phi}_k, \phi_l)$, by the number of field factors this contains, independently of the values of the fields positive integer exponents, r_i^α, s_i^x. The number of inequivalent monomial solutions rises fast with the order n considered, reaching orders of hundreds already at the relatively low order, $n = 4$. The solutions will be ordered according to increasing values of the number of field factors, $n = 1, 2, \cdots$, while allowing the power exponents to range over the discrete set of values, r_i^α, $s_i^x \in [0, 1, 2]$. Note that with this definition, the order n of a monomial is always bounded by its effective order, $\sum_{i=1}^{n} r_i^\alpha$ or $\sum_{i=1}^{n} s_i^x$. To avoid dealing with an overwhelming number of solutions, we generally restrict to orders $n \leq 4$.

The unbroken (anomaly free) Abelian symmetries $U(1)_x$ along a flat direction are identified by the kernel of the charge matrix, $A_{ai} = Q_a(\hat{\phi}_i)$, $[a = 1, N_a - 1]$ where $\hat{\phi}_i$ ranges over the modes excited along the flat direction and the index label a ranges over the anomaly free Abelian symmetries. In practice, we proceed by evaluating the left eigenvectors with zero eigenvalues of the matrix, A_{ai}, $[a, i = 1, \cdots, N_a - 1]$ namely, $x^T A = 0$, for some selected subset of the $\hat{\phi}_i$, and next retain among the Abelian charges, $Q_x = \sum_a x_a Q_a$, those yielding zero charges, $Q_x(\hat{\phi}_i) = 0$, for the full set of $\hat{\phi}_i$.

The effective trilinear interactions descending from non-renormalizable operators of order $n + 3$ are assigned string tree level coupling constants given by the approximate formula [57]

$$W_3^{(n)} = \tilde{c}_{ijk}^{(n)}(\phi)\Phi_i\Phi_j\Phi_k, \quad \left[\tilde{c}_{ijk}^{(n)}(\phi) = c_{ijk}^{(n)} < \prod_{i=1}^{n} \phi_i >, \; c_{ijk}^{(n)} \simeq g_s \frac{C_n I_n}{(\pi M_\star)^n} \right] \tag{II.12}$$

where C_n are constant coefficients of $O(1)$ and $I_n = \int \prod_{i=1}^{n} d^2 z_i f(z_i, \bar{z}_i)$ are integrals over the location of vertex operators on the world sheet sphere surface. The integrals for the four and five point string amplitudes evaluate numerically to [75–77]: $I_1 \simeq 70$, $I_2 \simeq 400$. The string coupling constant is commonly set at, $g_s = g_X/\sqrt{2} \simeq 1/2$. For the case of a 0-d flat direction, assigning the singlet fields the common VEV $\hat{\phi}$ determined by the Fayet-Iliopoulos term, one infers the following formula for the coefficients

$$|\hat{\phi}| \simeq \left(-\frac{\xi_A}{Q_A(\phi)} \right)^{\frac{1}{2}} \simeq \frac{g_s M_\star}{\pi} \left(-\frac{Trace(Q_A)}{192 Q_A(\hat{\phi})\sqrt{k_A}} \right)^{\frac{1}{2}} \implies \tilde{c}_{ijk}^{(n)} \simeq \frac{g_s^{n+1} C_n I_n}{(\pi^2\sqrt{192})^n} \left(-\frac{Trace(Q_A)}{Q_A(\hat{\phi})\sqrt{k_A}} \right)^{n/2}. \tag{II.13}$$

With increasing order of the non-renormalizable operators, we may thus anticipate a strong or mild suppression depending on the unknown extrapolation of I_n for large n and the model dependent size of the anomalous charges. Using, say, $I_n = 10^3$, $C_n = 1$, and dropping the factor depending on Q_A by setting, $\frac{-Trace(Q_A)}{\sqrt{k_A}Q_A(\hat{\phi})} \to 1$, gives, $\tilde{c}_{ijk}^{(n)} \simeq 10^{-2n} g_s^{1+n} C_n I_n \approx 10^{-2n+3}/2^{n+1}$. A similar formula holds for the μ-term coefficient of the electroweak Higgs bosons bilinear coupling, $\tilde{c}_{ij}^{(n)} h_i h_j$, by substituting, $\tilde{c}_{ij}^{(n)} = \mu^{(n)}/M_\star$. Bounding the coupling constant by the requisite ratio between Planck and effective supersymmetry breaking mass scales, $\mu^{(n)}/M_\star \leq \text{TeV}/M_\star \simeq 10^{-15}$, yields, $n \geq 8$, which shows that non-renormalizable effective operators starting from $O(\hat{\phi}^{(8)})h_i h_j$ can be tolerated. Considering instead the tentative extrapolation, $I_n \simeq 10^{\frac{n}{2}}$, $[C_n = 1]$, and the numerical estimates for the anomalous charge appropriate to model $A1$ (to be detailed below)

$$\left(\frac{-Trace(Q_A)}{\sqrt{k_A}Q_A(\hat{\phi})} \right)^{\frac{1}{2}} \simeq 10, \quad \frac{|\hat{\phi}|}{g_s M_\star} \simeq \frac{1}{\pi\sqrt{192}} \left(\frac{-Trace(Q_A)}{\sqrt{k_A}Q_A(\hat{\phi})} \right)^{\frac{1}{2}} \simeq 0.23, \tag{II.14}$$

then the corresponding condition on the μ-term coupling, $\mu^{(n)}/M_\star \simeq \frac{1}{2}10^{-n} \leq 10^{-15}$, gives the stronger bound on the order of non-renormalizable operators, $n \geq 14$. Note that lowering $\hat{\phi}_i$ should weaken the bounds on n.

III. SINGLET EXTENDED SUPERSYMMETRIC STANDARD MODELS

The Pati-Salam models $A1$, $A2$, B of [10] for the orbifold Z_{6-II} of T^6 with two Wilson lines only (since $W_2' = 0$), are described by the 16-d shift vectors for the gauge twist and Wilson lines of the $E_8 \times E_8$ group lattice:

$$\text{Model } A1: \; V = \frac{1}{6}(2,2,2,0^5)(1,1,0^6), \; W_2 = \frac{1}{6}(3,0^4,3^3)(0^8), \; W_3 = \frac{1}{6}(2,-2,0^6)(0^2,4,0^5).$$

$$\text{Model } A2: \ V = \frac{1}{6}(2^3, 0^5)(1^2, 0^6), \ W_2 = \frac{1}{6}(3, 0^4, 3^3)(0^8), \ W_3 = \frac{1}{6}(4, 2, -2, 0^5)(0, 4, 2^2, 0^4).$$

$$\text{Model } B: \ V = \frac{1}{6}(4, 1^2, 0^5)(2^2, 0^6), \ W_2 = \frac{1}{6}(0^3, 3^2, 0^3)(3, 0, 3, 0^5), \ W_3 = \frac{1}{6}(0, 2, -2, 0^5)(0^2, 4, 0^5). \quad \text{(III.1)}$$

The 4-d gauge groups for the above three models are:

$$\text{Model } A_1 \ : \ SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)^5 \times (SO(10)' \times SU(2)'). \quad \text{(III.2)}$$

$$\text{Model } A_2 \ : \ SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)^5 \times (SO(10)' \times SU(2)'). \quad \text{(III.3)}$$

$$\text{Model } B \ : \ SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)^6 \times SO(10)'. \quad \text{(III.4)}$$

Our results for the massless string spectra of models $A1$, $A2$ and B match to those of [10], except for certain assignments of oscillator numbers, $N^L_{I,\bar{I}}$, hence of R_I charges. The numerical data for the linear and cubic traces of the anomalous $U(1)_A$ gauge charge and the Fayet-Iliopoulos term, $\xi_A/(g_s M_\star)^2 \equiv Trace(Q_A)/(192\pi^2 \sqrt{k_A})$ are displayed in the following table. This also includes results for the total number of massless string states Φ_i and the degeneracies of the various modes.

Model	$Tr(Q_A)$	$Tr(Q_A^3)$	$Tr(Q_A Q_a^2)/2$	k_A	$\xi_A/(g_s M_\star)^2$	Φ_i	(S_i, Y_i)	h_i	C_i	f_i	$\bar{f}_i$	f_i^c	$\bar{f}_i^c$	$(q_i, \bar{q}_i)$
$A1$	-2	-0.00437	-0.16667	0.01750	-0.8×10^{-2}	78	34	7	9	3	0	5	2	4
$A2$	-131.4	-393.90	-10.94	24	-1.4×10^{-2}	77	36	3	3	3	0	4	1	6
B	13.66	0.312	1.139	0.182	1.7×10^{-2}	84	48	6	3	6	3	6	3	4

A. Model $A1$

The massless string spectrum of model $A1$ is displayed in Table I. The set of holomorphic invariant monomials of order $n \leq 2$ consist of eleven solutions, $P_\alpha = [Y_{11}S_8, \ S_3 S_6, \ Y_{14}Y_{18}^2, \ Y_{14}Y_{19}^2, \ Y_{15}Y_{19}^2, \ Y_{15}Y_{18}^2, \ Y_{18}^2, \ Y_{18}Y_{19}, \ Y_{18}^2 Y_{19}, \ Y_{18}Y_{19}^2, \ Y_{19}^2]$. None of these D flat directions is found to be lifted by the superpotential couplings in W_s at orders $n \leq 4$. Rather than analyzing each monomial P_α individually, we combine these multiplicatively into what we term as the 'composite' D-flat direction, $\mathcal{P}^{(2)} = P_1 \cdots P_{11}$, which then includes the eight modes, $Y_{11}, S_8, S_3, S_6, Y_{14}, Y_{18}, Y_{19}, Y_{15}$. The set of excluded couplings along the composite direction is clearly the union of all the sets excluded by the individual couplings of order n. All these directions are 0-d ones, hence with a common VEV given by, $|\hat{\phi}|/(g_s M_\star) \simeq 0.23$. The effective superpotential components obtained by assigning all the fields along $\mathcal{P}^{(2)}$ a common value $\hat{\phi}_i = \phi$ read at order $n \leq 4$

$$W_s^{(2)} \ = \ \phi S_1 S_{10} + \phi^3 Y_3 Y_9 + \phi^2 S_{11} S_{13} + \phi^2 S_{10} S_{15}.$$

$$W_h \ = \ h_1 h_1 [\phi^5 Y_{17} + \phi^5 S_4] + h_2 h_2 [\phi^4 S_{10}] + h_3 h_3 [S_1] + h_1 h_2 [\phi^2 S_{10}] + h_1 h_3 [\phi^4 + S_9] + h_2 h_3 [\phi^6 + \phi^4 S_9].$$

$$W_C \ = \ C_1 C_1 [\phi^3 Y_{10}] + C_2 C_2 [\phi^4 S_{10}] + C_3 C_3 [S_1] + C_4 C_4 [\phi^4 Y_7] + C_1 C_2 [0] + C_1 C_3 [\phi^2 Y_3] + C_1 C_4 [\phi Y_3 Y_9] \\ + C_2 C_3 [\phi^6 + \phi^4 S_9] + C_2 C_4 [\phi^5 Y_9] + C_3 C_4 [Y_5 S_{11}].$$

$$W_q \ = \ q_1 \bar{q}_1 [0] + q_1 \bar{q}_2 [\phi^5] + q_2 \bar{q}_1 [0] + q_2 \bar{q}_2 [S_{14}].$$

$$W_f \ = \ f_1^c \bar{f}_1^c [0] + f_1^c \bar{f}_2^c [S_2] + f_2^c \bar{f}_1^c [0] + f_2^c \bar{f}_2^c [\phi^2 Y_3] + f_3^c \bar{f}_1^c [S_{12}] + f_3^c \bar{f}_2^c [0].$$

$$\begin{aligned} W_{hff^c} \ = \ & h_1 f_1 f_1^c [1] + h_1 f_1 f_2^c [\phi Y_9] + h_1 f_1 f_3^c [\phi^2 Y_9] + h_1 f_2 f_1^c [0] + h_1 f_2 f_2^c [0] + h_1 f_2 f_3^c [0] \\ & + h_2 f_1 f_1^c [\phi^4] + h_2 f_1 f_2^c [\phi^3 Y_9] + h_2 f_1 f_3^c [\phi^4 Y_9] + h_2 f_2 f_1^c [0] + h_2 f_2 f_2^c [0] + h_2 f_2 f_3^c [0] \\ & + h_3 f_1 f_1^c [0] + h_3 f_1 f_2^c [0] + h_3 f_1 f_3^c [Y_7] + h_3 f_2 f_2^c [Y_1] + h_3 f_2 f_1^c [Y_1 + Y_2 + Y_4] + h_3 f_2 f_3^c [\phi^2]. \end{aligned} \quad \text{(III.5)}$$

The results for W_h through W_{hff^c} are illustrative ones in the sense that they include only a subset of the couplings at most linear or quadratic in the singlets which are not part of the flat direction. For convenience, we have listed the results for the bilinear couplings along $\mathcal{P}^{(2)}$ in the second column of Table II. The flavor bases for the massless fermions consist of the 3 modes $f_{1,2}$ and the 3 linear combinations of the 5 modes $f^c_{1,2,3}$ which are left unpaired. The fermion mass matrices are sums of 3×5 matrices $< h_k > f_{1,2} f^c_{1,2,3}$, $[k = 1, 2, 3]$ involving the combinations of 7 light bidoublets which acquire finite VEVs at the electroweak transition. The presence of an unsuppressed coupling $h_1 f_1 f_1^c$ suggests that h_1 should be the dominant component of the light electroweak doublets and f_1, f_1^c should be associated

TABLE I: Spectrum of string massless modes for the Pati-Salam model $A1$ of [10] with gauge group, $G_{422} \times (SO(10)' \times SU(2)') \times U(1)^5$. The pair of group factors $SU(2)_{1,2}$ identify with $SU(2)_{L,R}$. The non-Abelian singlet modes $\phi_i \sim (1,1,1)(1,1)'$ without and with oscillator excitations are denoted by S_i and Y_i. The non-singlet modes include the doublets, $d_i^l \sim (1,2,1)(1,1)'$, $d_i^r \sim (1,1,2)(1,1)'$, $d_i' \sim (1,1,1)(1',2')$; the bidoublets, $h_i \sim (1,2,2)(1,1)'$; the quartets, $q_i \sim (4,1,1)(1,1)'$, $\bar{q}_i \sim (\bar{4},1,1)(1,1)'$; the sextets, $C_i \sim (6,1,1)(1,1)'$; the bifundamentals, $f_i^c \sim (\bar{4},1,2)(1,1)'$, $\bar{f}_i^c \sim (4,1,2)(1,1)'$, $f_i \sim (4,2,1)(1,1)'$, $\bar{f}_i \sim (\bar{4},2,1)(1,1)'$, ; the decuplets, $T_i' \sim (1,1,1)(10',1')$; and the sextuplets, $\Sigma_i' \sim (1,1,1)(16',1')$, $\bar{\Sigma}_i' \sim (1,1,1)(\overline{16'},1')$. The first column lists the twisted sectors label g; the second column lists the modes name; the third column consists of three sub-columns which list the twisted subsector specified by the excited Wilson lines, with $N_{32} = 1$, $[g = 0]$; $N_{32} = 3n_2 + n_3 + 1$, $[g = 1]$; $N_{32} = n_3 + 1$, $[g = 2,4]$; and $N_{32} = n_2 + 1$, $[g = 3]$, the complex phase $\gamma(g)$ describing the orbifold twist action on the fixed points in the T_1^2 plane and the modes multiplicity (degeneracy) $\mathcal{D}$; the third column consists of three sub-columns which list the shifted H-momenta specified by the charges, $6R^I$, $[I = 1,2,3]$; and the fourth column consists of five subcolumns which list the Abelian gauge charges in the rotated basis starting with the anomaly free charges Q_a and ending with the single anomalous charge Q_A. In order to quote integer charges, the Abelian charges have been rescaled as, $Q_1/0.1667$, $Q_2/0.02083$, $Q_3/0.0104$, $4 \times Q_4/0.006944$, $Q_A/0.008333$. (Note that the first column entry for g is omitted in the right-hand part of the table.)

g	M	N_{32}	γ	$\mathcal{D}$	$6(R_1$	R_2	$R_3)$	Q_1	Q_2	Q_3	Q_4	Q_A	M	N_{32}	γ	$\mathcal{D}$	$6(R_1$	R_2	$R_3)$	Q_1	Q_2	Q_3	Q_4	Q_A
0	h_1	1	1	1	0	6	0	0	3	-3	-6	3	S_1	1	1	1	0	6	0	0	0	-6	24	-12
	T_1'	1	1	1	6	0	0	0	0	-3	12	-6	$\bar{f}_1^c$	1	1	1	0	0	6	3	3	0	0	0
	f_1	1	1	1	6	0	0	3	0	3	6	-3	f_1^c	1	1	1	0	0	6	-3	-3	0	0	0
1	Y_1	1	1	2	1	2	9	2	2	-1	4	-2	Y_2	1	1	2	1	2	-3	2	2	-1	4	-2
	Y_3	1	1	2	-5	-4	3	2	2	-1	4	-2	Y_4	1	1	2	-17	2	3	2	2	-1	4	-2
	Y_5	1	1	2	-5	2	3	2	-4	-1	4	-2	Y_6	1	1	2	-5	2	3	-4	-1	2	10	-5
	Y_7	1	1	2	-5	2	3	-4	-1	-4	-2	1	f_2^c	1	1	2	1	2	3	-1	-1	-1	4	-2
	f_2	1	1	2	1	2	3	-1	-1	-1	4	-2	Y_8	2	1	2	-5	2	3	4	1	-2	-10	-5
	d_1'	2	1	2	1	2	3	4	1	1	-4	7	Y_9	2	1	2	-5	2	3	-2	-2	1	-4	-8
	d_2'	2	1	2	1	2	3	-2	-2	4	2	4	Y_{10}	3	1	2	-5	2	3	0	3	0	18	1
	d_3'	3	1	2	1	2	3	0	3	3	-12	1	Y_{11}	3	1	2	-5	2	3	0	-3	0	18	1
	d_4'	3	1	2	1	2	3	0	-3	3	-12	1	$\bar{q}_1$	4	1	2	-5	2	3	-1	2	-1	4	-2
	q_1	4	1	2	1	2	3	-1	-1	2	10	-5	q_2	4	1	2	1	2	3	-1	-1	-4	-2	1
	d_1^l	4	1	2	1	2	3	2	2	2	10	-5	d_2^l	4	1	2	1	2	3	2	2	-4	-2	1
	d_1^r	4	1	2	1	-4	3	2	-1	-1	4	-2	d_2^r	4	1	2	-11	2	3	2	-1	-1	4	-2
	d_3^l	4	1	2	-5	2	3	-4	-1	-1	4	-2	$\bar{q}_2$	5	1	2	1	2	3	1	1	-2	-10	-5
	d_3^r	5	1	2	1	2	3	-2	1	1	-4	-8	d_4^l	5	1	2	1	2	3	-2	-2	-2	-10	-5
	d_4^r	6	1	2	-5	2	3	0	0	0	18	1	$h_1'^r$	6	1	2	1	2	3	0	0	3	-12	1
2	S_2	1	-1	1	2	4	0	4	4	-2	8	-4	Y_{12}	1	1	2	-4	4	0	4	-2	-2	8	-4
	Y_{13}	1	-1	1	2	10	0	4	-2	-2	8	-4	S_3	1	-1	1	2	4	0	4	-2	4	-16	8
	C_1	1	-1	1	2	4	0	-2	-2	-2	8	-4	f_3^c	1	1	2	2	4	0	1	1	-2	8	-4
	Y_{14}	2	1	2	-4	4	0	0	0	0	0	-10	Y_{15}	2	-1	1	2	10	0	0	0	0	0	-10
	T_1'	2	-1	1	2	4	0	0	0	0	18	-4	d_5'	2	1	2	2	10	0	0	0	3	6	2
	d_6'	2	-1	1	-4	4	0	0	0	3	6	2	S_4	2	-1	1	2	4	0	0	0	6	-24	2
	Σ_1'	2	1	2	2	4	0	0	0	3	-3	-1	S_5	2	-1	1	2	4	0	0	0	-6	-12	-4
	d_7'	2	1	2	2	4	0	0	0	-3	-6	8	S_6	3	-1	1	2	4	0	-4	2	-4	16	2
	d_8'	3	1	2	2	4	0	-4	2	-1	-14	2												
3	S_7	1	ω	2	3	0	3	0	3	-6	6	-3	S_8	1	ω^2	2	3	0	3	0	3	0	-18	9
	h_2	1	ω^2	2	3	0	3	0	0	-3	12	-6	h_3	1	1	4	3	0	3	0	0	3	-12	6
	C_2	1	ω^2	2	3	0	3	0	0	-3	12	-6	C_3	1	1	4	3	0	3	0	0	3	-12	6
	S_9	1	1	4	3	0	3	0	-3	0	18	-9	S_{10}	1	ω	2	3	0	3	0	-3	6	-6	3
4	C_4	1	1	2	4	2	0	2	2	2	-8	4	S_{11}	1	1	2	4	2	0	-4	2	-4	16	-8
	Y_{16}	1	1	2	4	-4	0	-4	2	2	-8	4	Y_{17}	1	-1	1	10	2	0	-4	2	2	-8	4
	S_{12}	1	1	2	4	2	0	-4	-4	2	-8	4	$\bar{f}_2^c$	1	-1	1	4	2	0	-1	-1	2	-8	4
	d_9'	2	-1	1	4	2	0	4	-2	1	14	-2	S_{13}	2	1	2	4	2	0	4	-2	4	-16	-2
	d_{10}'	3	-1	1	4	2	0	0	0	3	6	-8	S_{14}	3	1	2	4	2	0	0	0	6	12	4
	S_{15}	3	1	2	4	2	0	0	0	-6	24	-2	d_{11}'	3	1	2	10	2	0	0	0	-3	-6	-2
	d_{12}'	3	-1	1	4	-4	0	0	0	-3	-6	-2	T_2'	3	1	2	4	2	0	0	0	0	-18	4
	Y_{18}	3	1	2	4	-4	0	0	0	0	0	10	Y_{19}	3	-1	1	10	2	0	0	0	0	0	10
	$\bar{\Sigma}_1'$	3	-1	1	4	2	0	0	0	-3	3	1												

to the third generation fermions. The degenerate pairs of modes f_2 and f_2^c must then be associated to the first two generations. As long as the Dihedral D_4 discrete symmetry of [10, 86] is unbroken, the fermions mass matrices $f_2 f_i^c$ would include pairs of identical rows, which means that their rank is ≤ 2. This seemingly good starting point motivates one to search for mechanisms breaking the D_4 symmetry. As discussed in [10, 86], the non-renormalizable operators, $h_k f_i f_j^c \prod_{k,l,m,m'} S_k Y_l \mathcal{O}_{mm'}^c$, involving the composite singlet fields, $\mathcal{O}_{ij}^c = f_i^c \bar{f}_j^c$, produce contributions at the $PS \to SM$ transition which can improve predictions for the fermions mass matrices. We note that an alternative way to break the D_4 symmetry is by switching on the third Wilson line, $n_2' W_2'$.

That no constant or linear terms appears in $W_s^{(2)}$ indicates that all the $n = 2$ flat directions are indeed unlifted. The decoupled massive singlets are, $S_{1,10,11,13,15}$ and $Y_{3,9}$. The μ-term bilinear coupling in W_h is a rank 2 matrix with the massless mode, $(h_2 - \phi^2 h_1)$. However, the suppressed component is along h_1, which is the preferred mode whose VEV supposedly dominates the fermion masses. The trilinear couplings $W = (\phi^4 h_2 h_2 + \phi^2 h_1 h_2) S_{10}$ are seen to select the massive singlet S_{10}, which figures among the modes which acquire large masses along $\mathcal{P}^{(2)}$. However, it should be noted that S_{10} remains massless for a subset of the $n = 2$ directions P_α. The above result for W_f shows that no mass pairings among the matter modes are allowed. The mass matrices $M_{ij}^f f_i f_j^c$ have the single non-vanishing entry, $M_{11}^f = <h_1>$, if we omit the massive h_3 mode. Examination of the exotic modes mass couplings, shows that q_1, $\bar{q}_2$ and C_2, C_3 pair up, while the pairs q_2, $\bar{q}_1$ and C_1, C_4 remain massless. We also find that except for the coupling, $C_1 \bar{f}_2^c \bar{f}_2^c [\phi^2]$, all other trilinear couplings, $C_1 f_i^c f_j^c$, $C_1 \bar{f}_i^c \bar{f}_j^c$ in W_{Cff} are suppressed.

The 4×8 matrix of the Abelian charges for modes along $\mathcal{P}^{(2)}$ is found to have column rank 2. The existence of a secluded sector requires finding three singlets, ϕ_I, ϕ_J, ϕ_K, charged under the unbroken $U(1)_x$ with a mildly suppressed trilinear coupling, $c(\hat{\phi}_k) \phi_I \phi_J \phi_K$. A wide range of choices is clearly possible given the sizeable number of singlets orthogonal to the moduli space of $\mathcal{P}^{(2)}$, and the abundant number of monomial solutions in W_s. An inspection of the allowed couplings lets us select the following effective cubic or quartic order couplings $Y_1 Y_2 S_{12}$, $Y_8 S_7 S_{12} < S_{14} >$, $< Y_4^2 > Y_{12} Y_{16} S_{12}$, $Y_{10} S_7 S_{12} < S_{13} >$, that can give rise to a secluded sector with three massless singlets coupled by mildly suppressed trilinear interactions. A systematic search does not appear warranted at this stage.

The special role of S_{10} motivates us to examine its low order self-couplings, which are given for $n \leq 4$ by $W_s(S_{10}) = S_{10}[0] + S_{10}^2[\phi^3 S_7] + S_{10}^3[\phi^4 S_7^2]$. These resuls indicate that a cubic self-coupling, $S_{10}^3 \phi^4 < S_7^2 >$, cannot arise without an unacceptably large mass term, $S_{10}^2 \phi^3 < S_7 >$. Since this correlation between the quadratic and cubic couplings of S_{10} also holds for the individual flat directions, we conclude that the occurrence of a trilinear coupling, $h_i h_j S_{10}$, with a finite cubic self-coupling is not the favored option.

We also found useful to study a class of solutions on an individual basis. We present in Table II a subset of the bilinear couplings for four randomly selected F flat monomial solutions of order $n = 4$. The monomials $P_{II}^{(4)}$, $P_{III}^{(4)}$, $P_{IV}^{(4)}$ are seen to allow a pair of massless bidoublets, h_1, h_2, with preferred trilinear coupling, $W_h = h_1 h_2 S_{10}$. Along $P_{II}^{(4)}$, the diagonal trilinear coupling, $W_h = \phi^5 h_2 h_2 S_{13}$, is also present. The direction $P_I^{(3)}$ has all three bidoublets massless. However, some subsets of the exotic modes always remain massless.

To complete the present study, we have scanned the 70 and 48 flat monomial solutions of orders $n = 3$ and $n = 4$ in search of the bidoublets couplings of form, $W_h = h_i h_j (\mu_{ij} + \lambda_{ij}^k \phi^k)$. The global results combining the contributions from the various individual flat directions read:

- $n = 3$: $W_h = h_1 h_1[0] + h_2 h_2[\phi^4 S_{10}] + h_3 h_3[\phi S_{11}] + h_1 h_2[\phi^2 S_{13}] + h_1 h_3[\phi^2 Y_{15}] + h_2 h_3[\phi^6 + \phi^4 Y_{15}]$.
- $n = 4$: $W_h = h_1 h_1[0] + h_2 h_2[\phi^5 S_{12,13}] + h_3 h_3[\phi + \phi^2 Y_{12,14}] + h_1 h_2[S_{10} + \phi^2 Y_{6,11}]$
  $+ h_1 h_3[S_9 + \phi^5 Y_6] + h_2 h_3[\phi^4 Y_{12,14}]$. (III.6)

Happily, it appears that the cases with no trilinear couplings, $\mu_{ij} \neq 0$, $\lambda_{ij}^k = 0$, are outnumbered by those with no bilinear couplings, $\mu_{ij} = 0$, $\lambda_{ij}^k \neq 0$. The results for the directions $n = 3$ favor a scenario with a single light bidoublet h_1 having trilinear couplings, $\phi^2 h_1 h_2 S_{13}$ or $\phi^2 h_1 h_3 Y_{15}$. The results for the directions $n = 4$ favor three light bidoublets with canonical and non-canonical type trilinear couplings.

B. Model $A2$

The mass spectrum of model $A2$ is displayed in Table III. The D flatness conditions are solved at order $n = 2$ by the single monomial, $Y_9 S_8$, and at order $n = 3$ by the 7 monomials, $S_1 S_8^2 S_{15}$, $Y_9 S_8^2 S_{11}$, $S_3 Y_{16} Y_{18}$, $S_3 Y_{17} Y_{18}$, $S_3 Y_{17} Y_{19}$, $S_3 Y_{16} Y_{19}$, $S_8 S_{10} S_{15}$. The number of solutions at order $n = 4$ exceeds $O(100)$. The combined composite flat direction $\mathcal{P}^{(2)} \times \mathcal{P}^{(3)}$ for $n \leq 3$ includes the eleven modes, Y_9, S_8, S_1, S_{15}, S_{11}, S_3, Y_{16}, Y_{18}, Y_{19}, Y_{17}, S_{10}. This remains unlifted by all the singlet couplings in W_s of order $n \leq 4$. However, no unbroken $U(1)$ charges survive if the above eleven modes simultaneously acquire finite

TABLE II: Superpotential couplings for model $A1$ of bilinear order in h_i, C_i, q_i, $\bar{q}_i$ and orders $n \leq 4$ in the singlets. The column entries refer to the composite flat direction $\mathcal{P}^{(2)}$ and the four randomly selected individual flat directions: $P_I^{(3)} = Y_{12}Y_{16}Y_{19}^2$, $P_{II}^{(4)} = S_1 S_3 Y_{16} Y_{19}^2$, $P_{III}^{(4)} = S_1 S_3^2 S_4 S_6^2$, $P_{IV}^{(4)} = S_1 S_4 Y_{18}^2$. Empty entries correspond to cases where no coupling is present up to order $n = 4$.

W	$\mathcal{P}^{(2)}$	$P_I^{(3)}$	$P_{II}^{(4)}$	$P_{III}^{(4)}$	$P_{IV}^{(4)}$
$h_1 h_1$	$\phi^5 Y_{17} + \phi^5 S_4$				
$h_2 h_2$			$\phi^5 S_{13}$		
$h_3 h_3$	$S_1 + \phi S_{15} + \phi^5 S_7$	$\phi S_{11} + S_1$	$\phi + \phi^2 Y_{12}$	ϕ	$\phi + \phi^2 Y_{14}$
$h_1 h_2$	$S_{10} + \phi^2 S_{10}$	S_{10}	S_{10}	S_{10}	S_{10}
$h_1 h_3$	$\phi^4 + S_9 + \phi^3 S_{11}$	S_9	$S_9 + \phi^5 Y_6$	S_9	$S_9 + \phi^5 Y_{11}$
$h_2 h_3$	$\phi^6 + \phi^4 S_9$			$\phi^4 Y_{12} + \phi^4 Y_{14}$	
$C_1 C_1$	$\phi^3 Y_{10}$		$\phi^5 Y_3$		$\phi^5 Y_3$
$C_2 C_2$	$\phi^4 S_{10}$		$\phi^5 S_{13}$		
$C_3 C_3$	$S_1 + \phi S_{15} + \phi^5 S_7$	$\phi S_{11} + S_1$	$\phi + \phi^2 Y_{12}$	ϕ	$\phi + \phi^3 Y_{14}$
$C_4 C_4$	$\phi^4 Y_7$		$\phi^3 Y_5$		$\phi^3 Y_9$
$C_1 C_2$					
$C_1 C_3$	$\phi^2 Y_3 + \phi^4 Y_4$		$\phi^3 Y_3$		$\phi^3 Y_3$
$C_1 C_4$					
$C_2 C_3$	$\phi^6 + \phi^4 S_9$			$\phi^4 Y_{12} + \phi^4 Y_{14}$	
$C_2 C_4$	$\phi^5 Y_9$				
$C_3 C_4$			$\phi^2 Y_5$		$\phi^2 Y_9$
$q_1 \bar{q}_1$					
$q_1 \bar{q}_2$	$\phi^3 S_9 + \phi^5$			$\phi^4 Y_{14} + \phi^4 Y_{12}$	
$q_2 \bar{q}_1$					
$q_2 \bar{q}_2$	S_{14}	$\phi^2 S_{14}$	S_{14}	S_{14}	S_{14}

VEVs. The effective superpotential component W_s and those of bilinear and trilinear couplings obtained by assigning a common VEV ϕ to the modes $\hat{\phi}_i \in \mathcal{P}^{(2)} \times \mathcal{P}^{(3)}$ are given by

$$W_s = [\phi^3 S_9] + [\phi S_1 S_9 + \phi^2(Y_1^2 + Y_2^2 + Y_4^2) + \phi^2 Y_6 S_{17} + \phi^2 S_{13} S_{17} + \phi^2 Y_{10} S_{13}].$$

$$W_h = h_1 h_1[\phi^3 + \phi^3 Y_5] + h_2 h_2[0] + h_1 h_2[\phi^2 Y_3].$$

$$W_C = C_1 C_1[\phi^2 Y_3] + C_2 C_2[\phi Y_6] + C_1 C_2[\phi^2 Y_3].$$

$$W_q = q_1 \bar{q}_1[\phi^3 Y_1 + \phi^3 Y_2] + q_2 \bar{q}_2[\phi^3 Y_8] + q_3 \bar{q}_3[\phi Y_1 Y_7 + \phi Y_2 Y_7]$$
$$+ q_1 \bar{q}_2[\phi + \phi^2 S_9 + \phi^2 Y_{12}] + q_1 \bar{q}_3[\phi^4 Y_4] + q_2 \bar{q}_3[0] + \bar{q}_1 q_2[0] + \bar{q}_1 q_3[\phi^3 S_9 S_{17}] + \bar{q}_2 q_3[\phi^2 Y_1 Y_7].$$

$$W_f = f_1^c \bar{f}_1^c[\phi Y_3(Y_{12} + Y_{13} + S_{13})] + f_2^c \bar{f}_1^c[0].$$

$$W_{hff^c} = h_1 f_1 f_1^c[Y_3 S_4] + h_1 f_2 f_2^c[0] + h_1 f_1 f_2^c[0] + h_1 f_2 f_1^c[0] + h_2 f_1 f_1^c[0] + h_2 f_2 f_2^c[\phi Y_{10}] + h_2 f_1 f_2^c[0]$$
$$+ h_2 f_2 f_1^c[0].$$

$$W_{Cff} = C_1 f_1^c f_1^c[0] + C_1 f_2^c f_2^c[\phi^3 Y_{14}] + C_2 f_1^c f_1^c[\phi Y_{12}] + C_2 f_2^c f_2^c[\phi^2 Y_5]. \tag{III.7}$$

We see that the singlet superpotential W_s has no constant term but includes a harmless tadpole for the massive field, S_9. The massive singlets are $S_{1,9,13,17}$ and $Y_{1,2,4,6,10}$. The Higgs bidoublet couplings in W_h show that h_1 decouples, while h_2 is left massless but lacks trilinear couplings to singlets. Indeed, the quartic coupling, $\phi Y_{10} S_{13} \, h_2 h_2$, cannot generate an effective trilinear coupling at lower scales, since both of the singlets Y_{10}, S_{13} are massive. No large mass pairing takes place between the conjugate bifundamental modes. The suppressed couplings in W_{hff^c} preclude the possibility of discriminating between the light and heavy flavors. Some zero entries may be possibly lifted by assuming a finite VEV for the singlet Y_{10}. The sextet and quartet modes, except the pair, $q_1 \bar{q}_2$, remain massless. The couplings in W_{Cff} give no contributions to baryon number violating operators.

We have also scanned the individual D flat directions of orders $n = 2$, 3 and $n = 4$ in search of the bidoublet modes couplings. The full set of 15 monomials at orders $n = 2$, 3 are F flat. Of the 100 monomials at order $n = 4$, about

TABLE III: String spectrum of massless modes for model $A2$ of [10] of gauge group, $G_{422} \times (SO(10)' \times SU(2)') \times U(1)^5$, with the pair of group factors, $SU(2)_{1,2}$, identified with $SU(2)_{L,R}$. Same notational conventions as in Table I are used. The Abelian charges (given by integers except for Q_4) are defined in the rotated charge basis by the rescalings, $Q_1/0.1667$, $Q_2/0.1667$, $4\,Q_3/0.00260$, $Q_4/0.0007667$, $18\,Q_A/0.1244$.

g	M	N_{32}	γ	$\mathcal{D}$	$6(R_1$	R_2	$R_3)$	Q_1	Q_2	Q_3	Q_4	Q_A	M	N_{32}	γ	$\mathcal{D}$	$6(R_1$	R_2	$R_3)$	Q_1	Q_2	Q_3	Q_4	Q_A
0	Σ'_1	1	1	1	6	0	0	0	0	-18	15.05	-432	d'_1	1	1	1	6	0	0	0	0	0	-122.28	-54
	S_1	1	1	1	0	6	0	0	-6	48	0.63	-18	f_1	1	1	1	6	0	0	3	3	24	0.31	-9
1	Y_1	1	1	2	1	2	9	2	2	-20	-10.45	-96	Y_2	1	1	2	1	2	-3	2	2	-20	-10.45	-96
	Y_3	1	1	2	-5	-4	3	2	2	-20	-10.45	-96	Y_4	1	1	2	-17	2	3	2	2	-20	-10.45	-96
	Y_5	1	1	2	-5	2	3	2	-4	28	-9.82	-114	Y_6	1	1	2	-5	2	3	-4	-4	-20	-10.45	-96
	Y_7	1	1	2	-5	2	3	-4	2	28	-9.82	-114	f^c_1	1	1	2	1	2	3	-1	-1	4	-10.14	-105
	f_2	1	1	2	1	2	3	-1	-1	4	-10.14	-105	Y_8	2	1	2	-5	2	3	0	0	12	-91.55	-144
	Σ'_2	2	1	2	1	2	3	0	0	18	-15.05	36	d'_2	2	1	2	1	-4	3	0	0	12	30.73	-90
	d'_3	2	1	2	-11	2	3	0	0	12	30.73	-90	Y_9	2	1	2	-5	2	3	0	0	36	-30.10	468
	d'_4	3	1	2	1	2	3	4	-2	-4	-51.00	-120	Y_{10}	3	1	2	-5	2	3	4	-2	-4	71.28	-66
	d'_5	3	1	2	1	2	3	-2	4	-4	-51.00	-120	Y_{11}	3	1	2	-5	2	3	-2	4	-4	71.28	-66
	$\bar{q}_1$	4	1	2	-5	2	3	-1	2	-20	-10.45	-96	q_1	4	1	2	1	2	3	-1	-4	-20	-10.45	-96
	q_2	4	1	2	1	2	3	-1	2	28	-9.82	-114	d^l_1	4	1	2	1	2	3	2	-1	-44	-10.76	-87
	d^l_2	4	1	2	1	2	3	2	5	4	-10.14	-105	d^r_1	4	1	2	1	-4	3	2	-1	4	-10.14	-105
	d^r_2	4	1	2	-11	2	3	2	-1	4	-10.14	-105	d^l_3	4	1	2	-5	2	3	-4	-1	4	-10.14	-105
	h^r_1	5	1	2	1	2	3	0	3	-12	30.41	-81	h^l_1	5	1	2	1	2	3	0	-3	-12	30.41	-81
	$\bar{q}_2$	6	1	2	1	2	3	1	-2	-4	71.28	-66	d^l_4	6	1	2	1	2	3	-2	1	-28	70.97	-57
	d^r_3	6	1	2	1	2	3	-2	1	20	71.59	-75												
2	S_2	1	-1	1	2	4	0	4	4	-40	-20.90	-192	Y_{12}	1	1	2	-4	4	0	4	-2	8	-20.28	-210
	Y_{13}	1	-1	1	2	10	0	4	-2	8	-20.28	-210	S_3	1	-1	1	2	4	0	4	-2	32	41.18	402
	C_1	1	-1	1	2	4	0	-2	-2	8	-20.28	-210	f^c_2	1	1	2	2	4	0	1	1	-16	-20.59	-201
	d'_6	2	-1	2	2	4	0	-4	-4	-8	20.28	-186	S_4	2	1	1	2	4	0	-4	-4	16	-40.55	372
	S_5	3	-1	1	2	4	0	0	6	-24	60.83	-162	h_1	3	-1	1	2	4	0	0	0	-24	60.83	-162
	S_6	3	-1	1	2	4	0	0	-6	-24	60.83	-162	d'_7	3	1	2	2	4	0	0	0	24	-60.83	-234
	T'_1	3	-1	1	2	4	0	0	0	36	-30.10	72	Y_{14}	3	1	2	-4	4	0	0	0	24	61.45	-180
	Y_{15}	3	-1	1	2	10	0	0	0	24	61.45	-180	d'_8	3	1	2	2	4	0	0	0	48	0.63	378
3	S_7	1	ω	2	3	0	3	6	0	-12	-30.73	-306	S_8	1	ω^2	2	3	0	3	0	0	-36	30.10	324
	S_9	1	ω	2	3	0	3	0	6	-12	-30.73	-306	S_{10}	1	ω	2	3	0	3	0	-6	12	30.73	306
	S_{11}	1	1	4	3	0	3	0	0	36	-30.10	-324	S_{12}	1	ω	2	3	0	3	-6	0	12	30.73	306
	$\bar{q}_3$	2	ω^2	2	3	0	3	3	0	12	30.73	306	q_3	2	ω^2	2	3	0	3	-3	0	-12	-30.73	-306
	d^r_4	2	ω	4	3	0	3	0	-3	-12	30.41	315	d^r_5	2	1	2	3	0	3	0	3	12	-30.41	-315
4	C_2	1	1	2	4	2	0	2	2	-8	20.28	210	S_{13}	1	1	2	4	2	0	-4	2	-32	-41.18	-402
	Y_{16}	1	1	2	4	-4	0	-4	2	-8	20.28	210	Y_{17}	1	-1	1	10	2	0	-4	2	-8	20.28	210
	S_{14}	1	1	2	4	2	0	-4	-4	40	20.90	192	$\bar{f}^c_1$	1	-1	1	4	2	0	-1	-1	16	20.59	201
	d'_9	2	1	1	4	2	0	0	0	-48	-0.63	-378	T'_2	2	-1	2	4	2	0	0	0	-36	30.10	-72
	Y_{18}	2	-1	2	4	-4	0	0	0	-24	-61.45	180	Y_{19}	2	1	1	10	2	0	0	0	-24	-61.45	180
	d'_{10}	2	1	1	4	2	0	0	0	-24	60.83	234	S_{15}	2	-1	2	4	2	0	0	6	24	-60.83	162
	h_2	2	-1	2	4	2	0	0	0	24	-60.83	162	S_{16}	2	-1	2	4	2	0	0	-6	24	-60.83	162
	S_{17}	3	1	2	4	2	0	4	4	-16	40.55	-372	d'_{11}	3	-1	1	4	2	0	4	4	8	-20.28	186

80 are lifted by F-terms. The results obtained by combining the contributions from the various flat directions are

- $n = 2, \, 3 : \; W_h = h_1 h_1 [Y_{18}] + h_2 h_2 [0] + h_1 h_2 [0].$
- $n = 4 : \; W_h = h_1 h_1 [\phi^4 Y_{18,19}] + h_2 h_2 [\phi^4 Y_{14,15} + \phi^3 S_{13}] + h_1 h_2 [\phi^3 S_1 + \phi^2 Y_5 + \phi^4 S_2].$ \hfill (III.8)

We see again a clear trend in favor of several light bidoublets coupled by trilinear terms to massless singlets.

C. Model B

The massless spectrum for model B is displayed in Table IV. We find no holomorphic invariant monomials at order $n = 2$. At order $n = 3$, there appears 16 solutions of which a few representative monomials are: $P_\alpha = [S_{11} S_{18} Y_{25}, \; S_{11} S_{18} Y_{26}, \; S_{11} Y_{23} S_{20}, \; S_{11} Y_{24} S_{20}]$. The composite direction, $\mathcal{P}^{(3)}$, consists of the eight modes, $S_{11}, \; S_{18}, \; Y_{25}, \; Y_{26}, \; Y_{23}, \; S_{20}, \; Y_{24}, \; S_{15}$. No Abelian charges remain unbroken along $\mathcal{P}^{(3)}$. Assigning a common VEV ϕ to these modes yields the reduced superpotential

$$W_s \;=\; \phi Y_3 S_8 + \phi Y_4 S_5 + \phi S_3 Y_7 + \phi S_5 Y_5 + \phi S_6 Y_{10} + \phi S_8 Y_8.$$

$$W_h \;=\; h_1 h_1 [S_{16}] + h_2 h_2 [0] + h_3 h_3 [0] + h_1 h_2 [0] + h_1 h_3 [S_2] + h_2 h_3 [0].$$

$$W_C \;=\; C_1 C_1 [0] + C_2 C_2 [S_{16}] + C_1 C_2 [0].$$

$$W_q \;=\; q_1 \bar{q}_1 [\phi] + q_2 \bar{q}_2 [0] + q_1 \bar{q}_2 [\phi] + q_2 \bar{q}_1 [0].$$

$$
\begin{aligned}
W_f \;=\;& f_1 \bar{f}_1 [0] + f_1 \bar{f}_2 [0] + f_2 \bar{f}_1 [0] + f_2 \bar{f}_2 [\phi Y_3 + \phi Y_8] + f_1^c \bar{f}_1^c [S_1 (Y_{19} + Y_{20})] + f_1^c \bar{f}_2^c [0] \\
&+ f_2^c \bar{f}_1^c [S_1] + f_2^c \bar{f}_2^c [S_1 Y_{21}] + f_3^c \bar{f}_1^c [S_1 Y_{19}] + f_3^c \bar{f}_2^c [\phi^2 Y_1 Y_7].
\end{aligned}
$$

$$
\begin{aligned}
W_{hff^c} \;=\;& h_1 f_1 f_1^c [Y_{19}] + h_1 f_1 f_2^c [Y_{15} + Y_{15} Y_{19}] + h_1 f_1 f_3^c [\phi Y_{12} + \phi Y_{14}] \\
&+ h_1 f_2 f_1^c [0] + h_1 f_2 f_2^c [0] + h_1 f_2 f_3^c [0] + h_2 f_1 f_1^c [0] + h_2 f_1 f_2^c [0] + h_2 f_1 f_3^c [0] \\
&+ h_2 f_2 f_1^c [0] + h_2 f_2 f_2^c [0] + h_2 f_2 f_3^c [0] + h_3 f_1 f_1^c [\phi Y_{12} + \phi Y_{14}] + h_3 f_1 f_2^c [Y_{15}] + h_3 f_1 f_3^c [Y_{15}] \\
&+ h_3 f_2 f_1^c [0] + h_3 f_2 f_2^c [0] + h_3 f_2 f_3^c [0].
\end{aligned}
$$

$$
\begin{aligned}
W_{Cff} \;=\;& C_1 f_1 f_1 [0] + C_1 f_2 f_2 [0] + C_1 f_1^c f_1^c [Y_{19} + Y_{20}] + C_1 f_2^c f_2^c [Y_{15}] + C_1 f_3^c f_3^c [\phi Y_{12} + \phi Y_{14}] \\
&+ C_2 f_1 f_1 [0] + C_2 f_2 f_2 [0] + C_2 f_1^c f_1^c [0] + C_2 f_2^c f_2^c [0] + C_2 f_3^c f_3^c [0]. \hfill \text{(III.9)}
\end{aligned}
$$

The identical couplings for h_1 and C_2 is a consequence of the fact that these modes have same quantum numbers. The singlets superpotential W_s contains no constant or linear terms, while bilinear terms appear for the singlets $S_{3,5,6,8}$ and $Y_{3,4,5,7,8,10}$, which thus pick up large masses. The bidoublets couplings in W_h show that the three sets of h_i modes remain massless, and have trilinear couplings $(hhS_2 + hhS_{10})$ involving the massless singlets S_2 and S_{10}. No mass pairings arise between the conjugate bifundamental modes that would remove some of the f_i, f_i^c modes or decouple some of the mirror fermions, f_i, $\bar{f}_i$ and f_i^c, $\bar{f}_i^c$. The bidoublet-fermions couplings in W_{hff^c} give vanishing mass matrices for the matter fermions. The flavor structure is problematic since it gives no hint on how to discriminate between the heavy and light flavors or between the matter and Higgs modes, independently of the dominant linear combinations of the bidoublets. The results for W_C and W_q show that all the sextet modes, C_i, and the single pair of quartet modes, q_2, $\bar{q}_1$, remain unpaired.

The special role of the singlet S_{16} motivates us in studying its self-couplings at orders $n \le 4$. The results for $\mathcal{P}^{(3)} : \; W_s(S_{16}) = S_{16}[0] + S_{16}^2[0] + S_{16}^3[0]$, indicate that the mode S_{16} is light but has a suppressed cubic coupling. The results for $\mathcal{P}^{(4)}$, $W_s(S_{16}) = S_{16}[0] + S_{16}^2[\phi^5 Y_{13} + \phi^5 Y_{11}] + S_{16}^3[\phi^4 Y_3 + \phi^4 Y_8]$ would also allow for a light mode S_{16} provided that the massive singlets $Y_{3,8,11}$ acquire small VEVs.

Results for seven randomly selected individual flat directions are displayed in Table V. For all monomials other than P_{IV}, we see that all the h_i remain light and have trilinear couplings to singlets, but that the sextets and a subset of the quartets fail to decouple. We have also performed global type scans of the D flat monomial solutions restricted to the bidoublets couplings. Of the 16 and 100 solutions at orders $n = 3$ and 4, we find 0 and 10 monomials lifted by F-terms. The bidoublets couplings combining the contributions from the various flat directions are given by

- $n = 3 : \; W_h = h_1 h_1 [S_{16}] + h_2 h_2 [0] + h_3 h_3 [0] + h_1 h_2 [0] + h_1 h_3 [S_2] + h_2 h_3 [0].$
- $n = 4 : \; W_h = h_1 h_1 [S_{16} + \phi Y_{19} + \phi Y_{20}] + h_2 h_2 [0] + h_3 h_3 [\phi Y_{15} + \phi S_2] + h_1 h_2 [0]$
  $\qquad\qquad + h_1 h_3 [\phi + \phi S_2 + \phi S_{16}] + h_2 h_3 [\phi^3 Y_{13} + \phi^2 S_5 + \phi^2 Y_1].$ \hfill (III.10)

TABLE IV: Massless string modes for model B of [10] with gauge group, $G_{422} \times (SO(10)') \times U(1)^6$. We use same notational conventions as in Table I, except that the pair of group factor $SU(2)_{1,2}$ identify here with $SU(2)_{R,L}$. The quoted integer rescaled charges are related to the initial ones by the rescalings, $4\,Q_1/0.005208$, $25\,Q_2/0.002042$, $12\,Q_3/0.009615$, $9\,Q_4/0.00347$, $Q_5/0.000672$, $Q_A/0.017796$.

g	M	N_{32}	γ	$\mathcal{D}$	$6R_1$	R_2	R_3	Q_1	Q_2	Q_3	Q_4	Q_5	Q_A	M	N_{32}	γ	$\mathcal{D}$	$6R_1$	R_2	R_3	Q_1	Q_2	Q_3	Q_4	Q_5	Q_A
0	S_1	1	1	1	0	6	0	48	210	−12	−24	−12	−12	T_1'	1	1	1	0	6	0	0	0	0	−81	6	6
	C_1	1	1	1	0	6	0	−12	−180	−12	−24	−12	−12	f_1^c	1	1	1	0	0	6	30	195	0	0	0	0
	f_2^c	1	1	1	6	0	0	−18	−15	12	24	12	12	$\bar f_1^c$	1	1	1	0	0	6	−30	−195	0	0	0	0
1	Y_1	1	1	2	1	−4	3	16	155	32	−17	7	7	Y_2	1	1	2	−11	2	3	16	155	32	−17	7	7
	S_2	1	1	2	1	2	3	16	155	−46	70	4	4	h_1	1	1	2	1	2	3	4	−25	20	−41	−5	−5
	C_2	1	1	2	1	2	3	4	−25	20	−41	−5	−5	Y_3	1	1	2	−5	2	3	4	−280	29	−23	4	4
	Y_4	1	1	2	−5	2	3	−44	20	23	−35	−2	−2	S_3	2	1	2	1	2	3	32	55	34	−13	71	7
	S_4	2	1	2	1	2	3	32	55	34	68	−28	4	S_5	2	1	2	1	2	3	32	55	−44	−7	−19	13
	Y_5	2	1	2	−5	2	3	−28	−80	25	−31	62	−2	Y_6	2	1	2	−5	2	3	−28	−80	25	50	−37	−5
	Y_7	2	1	2	−5	2	3	−28	−80	−53	−25	−28	4	S_6	3	1	2	1	2	3	0	255	30	−21	−57	7
	S_7	3	1	2	1	2	3	0	255	30	60	30	−2	S_8	3	1	2	1	2	3	0	255	−48	−15	39	7
	Y_8	3	1	2	−5	2	3	−12	−180	27	−27	−60	4	Y_9	3	1	2	−5	2	3	−12	−180	27	54	27	−5
	Y_{10}	3	1	2	−5	2	3	−12	−180	−51	−21	36	4	d_1^l	4	1	2	1	2	3	16	155	−7	−14	−38	10
	d_1^r	4	1	2	−5	2	3	4	−25	−19	−38	−50	−2	d_2^r	4	1	2	1	2	3	4	−25	−19	43	37	−11
	$\bar q_1$	4	1	2	1	2	3	−14	−40	−7	−14	−38	10	d_2^l	5	1	2	1	2	3	32	55	−5	−10	26	10
	d_3^r	5	1	2	1	2	3	−16	100	−2	−4	29	13	d_4^r	5	1	2	−5	2	3	20	−125	−17	−34	14	−2
	d_3^l	5	1	2	−5	2	3	−28	−80	−14	−28	17	1	q_1	5	1	2	−5	2	3	2	115	−14	−28	17	1
	$\bar q_2$	5	1	2	1	2	3	2	−140	−5	−10	26	10	d_5^r	6	1	2	1	2	3	−12	75	−21	39	−27	−11
	d_4^l	6	1	2	1	2	3	−12	−180	−12	57	−18	−2	q_2	6	1	2	1	2	3	18	15	−12	57	−18	−2
2	h_2	1	−1	1	2	4	0	20	130	−26	29	−1	−1	S_9	1	1	2	2	4	0	20	−125	61	−40	11	11
	S_{10}	1	1	2	2	4	0	−28	175	55	−52	5	5	S_{11}	1	−1	1	2	4	0	8	−50	40	−82	−10	−10
	f_1	1	1	2	2	4	0	−10	−65	−26	29	−1	−1	S_{12}	2	1	2	2	4	0	−12	75	57	−48	69	5
	Y_{11}	2	1	2	2	10	0	−12	75	−21	−42	−21	11	Y_{12}	2	−1	1	−4	4	0	−12	75	−21	−42	−21	11
	T_2'	2	1	2	2	4	0	−12	75	−21	39	−27	5	S_{13}	2	−1	1	2	4	0	24	−150	−36	−72	−36	−4
	S_{14}	3	1	2	2	4	0	4	−25	59	−44	−53	11	Y_{13}	3	1	2	2	10	0	4	−25	−19	−38	43	11
	Y_{14}	3	−1	1	−4	4	0	4	−25	−19	−38	43	11	T_3'	3	1	2	2	4	0	4	−25	−19	43	37	5
	$\bar\Sigma_1'$	3	1	2	2	4	0	4	−25	20	−1	−8	8	S_{15}	3	−1	1	2	4	0	−8	50	−40	−80	22	−10
3	Y_{15}	1	1	4	−3	0	3	24	105	−6	−12	−6	−6	Y_{16}	1	ω	2	9	0	3	24	105	−6	−12	−6	−6
	Y_{17}	1	ω^2	2	3	0	9	24	105	−6	−12	−6	−6	Y_{18}	1	ω^2	2	3	0	−3	24	105	−6	−12	−6	−6
	Y_{19}	1	1	4	3	0	9	−24	−105	6	12	6	6	Y_{20}	1	1	4	3	0	−3	−24	−105	6	12	6	6
	Y_{21}	1	ω	2	−3	0	3	−24	−105	6	12	6	6	Y_{22}	1	ω^2	2	9	0	3	−24	−105	6	12	6	6
	f_2	1	1	4	3	0	3	6	90	6	12	6	6	f_3^c	1	1	4	3	0	3	6	90	6	12	6	6
	$\bar f_1$	1	ω^2	2	3	0	3	−6	−90	−6	−12	−6	−6	$\bar f_2^c$	1	ω^2	2	3	0	3	−6	−90	−6	−12	−6	−6
	d_6^r	2	ω	4	3	0	3	24	−150	−36	9	−42	6	d_7^r	2	1	2	3	0	3	−24	150	36	−9	42	−6
4	S_{16}	1	1	2	4	2	0	−8	50	−40	82	10	10	S_{17}	1	−1	1	4	2	0	28	−175	−55	52	−5	−5
	S_{18}	1	−1	1	4	2	0	−20	125	−61	40	−11	−11	h_3	1	1	2	4	2	0	−20	−130	26	−29	1	1
	$\bar f_2$	1	−1	1	4	2	0	10	65	26	−29	1	1	S_{19}	2	1	2	4	2	0	8	−50	40	80	−22	10
	T_4'	2	−1	1	4	2	0	−4	25	19	−43	−37	−5	Y_{23}	2	1	2	10	2	0	−4	25	19	38	−43	−11
	Y_{24}	2	−1	1	4	−4	0	−4	25	19	38	−43	−11	S_{20}	2	−1	1	4	2	0	−4	25	−59	44	53	−11
	Σ_1'	2	−1	1	4	2	0	−4	25	−20	1	8	−8	S_{21}	3	1	2	4	2	0	−24	150	36	72	36	4
	T_5'	3	−1	1	4	2	0	12	−75	21	−39	27	−5	Y_{25}	3	1	2	10	2	0	12	−75	21	42	21	−11
	Y_{26}	3	−1	1	4	−4	0	12	−75	21	42	21	−11	S_{22}	3	−1	1	4	2	0	12	−75	−57	48	−69	−5

TABLE V: Bilinear superpotential couplings for model B in the modes h_i, C_i, q_i, $\bar{q}_i$ of order, $n \leq 4$ in the singlets. The column entries refer to the three flat directions of order $n = 3$: $P_I = S_{11}S_{18}Y_{25}$, $P_{II} = S_{11}Y_{23}S_{20}$, $P_{III} = S_{15}Y_{23}Y_{26}$, and the four randomly selected flat directions of order $n = 4$: $P_{IV} = S_2Y_6S_{11}^2S_{20}$, $P_V = Y_4^2Y_{15}^2S_{17}Y_{26}$, $P_{VI} = Y_6S_{15}Y_{15}Y_{26}$, $P_{VII} = Y_9S_{15}Y_{18}Y_{23}$. Empty entries correspond to cases where no coupling is present up to order $n = 4$.

W	P_I	P_{II}	P_{III}	P_{IV}	P_V	P_{VI}	P_{VII}
h_1h_1	S_{16}	S_{16}	S_{16}	$S_{16}+\phi Y_{19,20}$	S_{16}	S_{16}	S_{16}
h_2h_2							
h_3h_3				ϕY_{15}	$\phi S_2 + \phi^2 S_{16}$	$\phi S_2 + \phi^2 S_{16}$	
h_1h_2							
h_1h_3	S_2	S_2	S_2	ϕ	$S_2+\phi S_{16}$	$S_2+\phi S_{16}$	S_2
h_2h_3				$\phi^3 Y_{13}$	$\phi^2 Y_1 + \phi^2 S_5$		
C_1C_1							
C_2C_2	S_{16}	S_{16}	S_{16}	$S_{16}+\phi Y_{19,20}$	S_{16}	S_{16}	S_{16}
C_1C_2							
$q_1\bar{q}_1$	ϕ	Y_{25}	$Y_{25}+\phi^2 Y_{11,14}+\phi^2 S_{19}$	Y_{25}			
$q_2\bar{q}_2$							
$q_1\bar{q}_2$	Y_{23}	ϕ		$\phi+\phi^2 Y_{12}$	$Y_{25}+\phi^2 Y_{11}+\phi^2 S_{10}$	$Y_{25}+\phi^2 Y_{11}+\phi^2 S_{19}$	$Y_{25}+\phi^3 Y_{22}+\phi^4 Y_{19,20}$
$q_2\bar{q}_1$				Y_{23}	Y_{23}	Y_{23}	$\phi+\phi^2 Y_{21}$

These results again favor the scenario in which light bidoublets interact by trilinear couplings to singlets. A scan of the flat directions reveals the existence of a large majority of individual monomial directions along which the mode S_{16} is light but with a suppressed cubic self-coupling.

IV. DISCUSSION AND CONCLUSIONS

Let us first state some general features of our results. The top-down construction for the Z_{6-II} orbifold has a large number of flat directions and a rich structure of superpotential couplings. This contrasts with the situation prevailing for the Z_3 orbifold models [1, 7, 35], but is in harmony with that in non-prime orbifolds or free fermions. Comparing with the string spectra obtained in intersecting brane models [69, 78] would not be very teaching because the existing type I string models are more akin to bottom-up type constructions.

We have carried the F flatness test indiscriminately for both types A and B flat directions, restricting to the superpotential monomials with at most four distinct singlet field factors, $W_s(\phi_i) = \prod_{i=1}^{n} \phi_i^{s_i}$, $[n \leq 4, \ s_i \leq 2]$. These contain a subset of the monomials of absolute order $\leq 2n = 8$. It is intuitively clear that the F flatness condition is more severe for the lower order superpotential monomials, since these are more likely to contain all (or all but one) of the fields excited along the flat direction. Upon pushing the F flatness test in model B to the order $n = 5$ of W_s, we find that out of 100 order $n = 4$ flat monomials, 34 are lifted by the 3 monomials of order $n = 3$ in $W_s = S_1Y_{19}^2$, $Y_4S_5Y_{25}$, $S_5Y_5Y_{23}$. Including all the monomials in W_s up to order 8, selects, 0, 6, 40 flat monomials of orders $n = 2, \ 3, \ 4$.

To determine which of the singlets decouple and which remain massless, we have derived the mass matrices for the S_i and Y_i by scanning over each individual flat direction up to absolute order 8. We found that one cannot decouple all the singlets at the string scale by these means in any of the models. However, taking the combinations of all the singlets of a given order, we found that all singlets of models $A1$ and $A2$ can be decoupled if the order is large enough, $n \geq 5$ and 7 respectively, and the number of fields involved is itself large enough. One might prefer vacua defined by smaller sets of fields, and indeed our aim here is not to decouple all the singlets but leave a few of them massless in order to obtain an NMSSM like model. We thus prove that there is room for decoupling all but a few relevant modes, should one perform a thorough scan over the flat directions. As far as model B is concerned, the number of singlet fields is too high to decouple all of them at the order considered, and making this model viable on a phenomenological basis would require giving a lot of singlet modes a mass at the level of the Pati-Salam breaking down to the Standard Model gauge group.

Our study of the fermion mass generation has been rather sketchy. The preferred candidate for the electroweak Higgs bidoublet is obvious only in model $A1$. To discriminate between the heavy and light flavors in models $A2$ and B requires a closer analysis of the couplings of bidoublets and bifundamentals. Such a task would be warranted once one has really in hand a benchmark type model. All three models satisfy at the string mass scale a D_4 family symmetry with a number of bifundamental modes f or f^c from the twisted sectors T_1 or T_3 transforming as doublets.

An acceptable description of the fermions flavor structure can be achieved only if the D_4 symmetry is broken. On side of the promising mechanism using the condensation at the Pati-Salam breaking scale of the composite singlet fields, $\mathcal{O}^c_{ij} = f^c_i \bar{f}^c_j$, stringy mechanisms can be envisionned to lift the degeneracy of the modes in the $T_{1,3}$ sectors. One could use blow up submanifolds of different sizes at the two fixed points, or consider the so far lightly explored models with three Wilson lines [34].

The existence of a light pair of Higgs bosons seems to be correlated with the presence of massless exotic color triplets descending from sextet and bifundamental modes, C_i, f'^c_i. This is unavoidable in models $A1$ and B because of certain h_i and C_i having identical quantum numbers. A single pair of q_i, $\bar{q}_i$ modes always fails to decouple. Since the exotic modes are vector like, they could pair off or couple to lighter modes into which they decay, through lower scale physics, including the $PS \to SM$ transition. We have not examined here the decoupling of the weak doublet and singlet exotic modes, $d^l = (N^l, E^l)$, $d^r = (E^r, N^r)$, since these represent perhaps a lesser threat on the low energy theory.

Our main purpose in this study was to establish an existence proof, based on three representative string models, of supersymmetric models where suppressed bilinear couplings of the electroweak Higgs doublets coexist with unsuppressed trilinear couplings to singlets, $\mu h_k h_l + \lambda h_k h_l \phi$, $[\mu \le O(\phi^8),]$. Rather than scanning over the flat directions, we have followed an approximate procedure making use of what we term as composite flat directions. This is admittedly open to criticism, especially at high orders, because of the large number of excited singlets. However, the qualitative orientation this approach gives does not seem invalidated by selecting randomly, or scanning in a global way, a subset of individual flat directions. We frequently encounter canonical and non-canonical type trilinear couplings, $\lambda^k_{ij} h_i h_j \phi_k$, however, without any obvious correlations between these bidoublets and those dominating the fermion-Higgs Yukawa couplings.

Additional $U(1)'$ gauge symmetries are generally present in the individual flat directions of order $n \le 4$, but they are absent in most composite directions. It is difficult to decide if a gaugino supersymmetry breaking mediation or a secluded sector scenario is favored since our searches give no clue on the size of the Z' boson mass scale and the $Z - Z'$ mixing angle.

In summary, the search of models with extra singlets is considerably facilitated in the Z_{6-II} orbifold by the rich mini-landscape of vacua. We find a clear preference towards an active role for extra singlets, but the presence of sizeable trilinear couplings $hh\phi$ along with strongly suppressed bilinear couplings hh is not systematic. The study of solutions was made tractable thanks to the restriction to low orders, $n \le 4$. Beyond this order one must consider representative samples of the flat directions. However, the composite flat directions of low orders, $n \le 4$, seem to capture general features not invalidated by global scans of the individual directions, as confirmed by the similar conclusions for models $A1$ and B. The strong $O(\phi^8)$ suppression required for the μ couplings suggests, however, that definitive conclusions could only be made until the searches include higher order couplings. Weak and strong points are present in all three models, with no model faring best on all issues.

Appendix A: Flat directions in heterotic string compactification

Given a gauge theory with the Lie group G_a and superpotential W, which includes the set of massless chiral supermultiplets, ϕ_i, $[i = 1, \cdots, N]$ then the D flatness conditions are given by: $0 = D_a = (\phi^\star_i (T^a)_{ij} \phi_j)$, where T_a are the Lie algebra generators, and the F flatness conditions are given by: $W_i \equiv \frac{\partial W}{\partial \phi_i} = 0$. In the scalar fields ϕ_i vector space, the D flat directions are parameterized by the gauge invariant monomials, $P_\alpha(\phi_i)$, satisfying the equations, $P_{\alpha,i} \equiv \frac{\partial P_\alpha}{\partial \phi_i} = c_\alpha \phi^\star_i$, for suitably chosen complex constants, c_α, with the ϕ_i substituted by their (constant) VEV [1, 80]. (This statement is easily checked by writing the G_a gauge invariance condition as, $0 = \delta_a P_\alpha = \sum_i \frac{\partial P_\alpha}{\partial \phi_i} \delta_a \phi_i = c_\alpha \sum_i \phi^\star_i (T_a \phi)_i$.) For an anomalous gauge symmetry [81–83] $U(1)_A$ with the ϕ_i charges, $Q^i_A = Q_A(\phi_i)$, and a finite charge anomaly implying the presence of a Fayet-Iliopoulos term with coupling constant ξ_A, the D-flatness condition is modified to

$$0 = D_A = \sum_i \phi^\star_i Q^i_A \phi_i + \xi_A, \quad \left[\xi_A = e^{2\phi} M^2_\star \delta^A_{GS} = \frac{g^2_s M^2_\star}{192\pi^2} \frac{1}{\sqrt{k_A}} Tr(Q_A), \; k_A = 2 \sum_I (Q^I_A)^2 \right] \tag{A.1}$$

where $g_s = <e^\phi>$ is the string coupling constant, with ϕ the 10-d dilaton field, and Q_A^I, $[I = 1, \cdots, 16]$ are the orthogonal basis components of the anomalous charge generator in the $E_8 \times E_8$ group weight lattice. For the set of gauge group factors, $G = (\prod_a G_a) \times U(1)_A$, the D flatness conditions is solved by considering holomorphic invariant monomials, $P_\alpha(\phi_i)$, uncharged with respect to the anomaly free gauge factors G_a, but with finite anomalous $U(1)_A$ charge, $Q_A(P_\alpha) \neq 0$, of opposite sign to $\xi_A \propto Trace(Q_A)$. In the supergravity context, the F flatness equations are, $W = 0$, $F_i = \partial W / \partial \phi_i = 0$, where W is built from holomorphic monomials invariant under the complete set of non-Abelian and Abelian gauge symmetries, including the global type string theory symmetries.

Most of our applications deal with the field space of non-Abelian singlets charged under the Abelian gauge groups, $G = (\prod_{a=1}^{N_a} U(1)_a) \times U(1)_A$. The D-flat directions are parameterized by the monomials, $P_\alpha = \prod_i \phi_i^{r_i^\alpha}$, solving the simultaneous equations: $Q_a(P_\alpha) = 0$, $Q_A(P_\alpha) \neq 0$ of sign opposite to ξ_A. By contrast, the superpotential couplings are constructed from the gauge invariant monomials with respect to both non-Abelian and Abelian (anomaly free and anomalous) gauge groups which obey the string selection rules, listed for the orbifold Z_{6-II} in Eqs. (B.10). The submanifold of vacua for non-Abelian singlets [55] is parameterized by the holomorphic invariant monomials, $P_\alpha = \prod_i \phi_i^{r_i^\alpha}$, $[r_i^\alpha \geq 0]$ described by the column vectors, $\vec{r}^\alpha = (r_1^\alpha, \cdots, r_N^\alpha) \in Z_+^N$, subject to the equations,

$$Q_a(P_\alpha) = \sum_i \frac{\partial P_\alpha}{\partial \phi_i}(Q_a \phi)_i = P_\alpha(\phi) \times \sum_i r_i^\alpha Q_a^i = 0, \ Q_A(P_\alpha) = P_\alpha(\phi) \times \sum_i r_i^\alpha Q_A^i = -c_A \xi_A, \ [c_A > 0]. \ \text{(A.2)}$$

These equations are solved by the simple solution, $|\phi_i|/\sqrt{r_i^\alpha} = |\phi| = 1/c_A$. The flat directions may have 0, 1 or more dimensions, where the 0-d case involves a fixed common VEV, $\phi = (-\xi_A / \sum_i Q_A^i r_i^\alpha)^{\frac{1}{2}}$, and the 1-d or higher dimension cases involve a single or more free complex parameters. When the system of linear equations is underdetermined, the solutions depend on free continuous parameters whose number identifies with the moduli space dimension, $\mathcal{D} \leq N - rank(A)$. The vector space of column vectors, $\vec{r}$, is generated by the basis of column vectors, $\vec{r}^A = (r_1^A, \cdots, r_N^A)$, associated to the holomorphic invariant monomials, $M_A = \prod_i \phi_i^{r_i^A}$, $[A = 1, \cdots, \mathcal{D}]$ such that any solution can be written as, $P^n = \prod_\alpha M_A^{n_A}$, for $n \geq 1$ and $n_A \in Z$ (positive or negative signs). More conveniently, one can also consider the superbasis of one-dimensional invariant monomials, P_α, satisfying the condition that they cannot be factorized into products of two or more invariant monomials.

The D flat direction described by the monomial solution, $P_\alpha(\phi) = \prod_i \phi_i^{r_\alpha^i}$, can be lifted by F-terms of type A, $W_s^A = (\prod'_{i \in P_\alpha} \phi_i^{r^i})^n$ or type B, $W_s^B = \Phi(\prod'_{i \in P_\alpha} \phi_i)^n$, $[\Phi \notin P_\alpha]$ consisting of gauge invariant superpotential monomials with all, or all but one, field factors included in $P_\alpha(\phi)$. In the terminology of [56, 57, 62], these refer to the gauge invariant non-renormalizable couplings in the field theory action, prior to applying the string selection rules. For the F flatness to remain valid to arbitrary orders of the full superpotential, an infinite number of conditions must be imposed for a type A direction but a finite number for a type B direction. The F flatness conditions are, of course, more restrictive when set on the string theory action.

Appendix B: Review of string construction for Z_{6-II} orbifold

The orbifold Z_{6-II} is described by the rotation vector, $\Theta^I = e^{2i\pi v_6^I}$, or the twist vector, $v_6^I = \frac{1}{6}(1, 2, -3)$, acting on the complexified coordinate and fermion field components X^I, ψ^I associated to the complex planes of T_I^2. This orbifold is equivalent to the direct product of suborbifolds, $Z_2 \times Z_3$, with twist vectors, $v_2^I = 3v_6^I = \frac{1}{2}(1, 0, -1)$, $v_3^I = 2v_6^I = \frac{1}{3}(1, -1, 0)$. The anisotropic compactifications [10, 12] are realized with 6-d tori which factorize on the three maximal tori T_I^2 of the rank 2 semi-simple groups, G_2, $SU(3)$, $SO(4)$, with lattice basis vectors $e_{1,2}$, $e_{3,4}$, $e_{5,6}$, and orbifold point group action represented by the 2×2 matrices for the Coxeter operators $C_I^T \simeq \Theta^I$ in the lattice bases,

$$\Theta(G_2) = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}, \ \Theta(SU(3)) = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \ \Theta(SO(4)) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{B.1}$$

The input data for the orbifold fixed points and shift vectors in the various twisted sectors is displayed in Table VI. The shift vectors $u_{g,f} \in \Lambda$, associated to the fixed points f in sectors $T_g \sim \Theta^g$, are defined by, $(1-\Theta^g)f = u_{g,f}$, modulo elements of the sublattice, $\Lambda_{\Theta^g} = (1-\Theta^g)\Lambda$, of the torus lattice Λ. The shift vectors are in 1-to-1 correspondence with the orbifold space group elements, $(\Theta^g, u_{g,f})$, which act on the orthogonal frame coordinates as, $X \to \Theta^g X + u_{g,f}$. Each shift vector $u_{g,f}$ represents a conjugacy class element of the torus lattice coset, $\Lambda/\Lambda_{\Theta^g}$. The fixed points f, shift

TABLE VI: Geometric data for the orbifold Z_{6-II} on the torus $T^6 = T_1^2 \times T_2^2 \times T_3^2$ of lattice, $\Lambda(G_2) + \Lambda(SU(3)) + \Lambda(SO(4))$. The line entries refer to the twisted sectors, T_g, $[g = 1, \cdots, 4]$ of Z_{6-II} with $T_1 \sim T_5$, and the associated T_{g_1,g_2} sectors of the orbifold, $Z_2 \times Z_3$. The second colum displays the number of fixed points $\mathcal{N}_{fp}^I$ in the three complex planes T_I^2. The third column consists of three subcolumns which display for the lattice $\Lambda(G_2)$, the fixed points and shift vectors, f_a, u_a and f_{ab}, u_{ab}, for the elements $\Theta^{2,4}$ and Θ^3, the linear combination eigenvectors, $|\gamma(g)>$, and the eigenvalues $\gamma(g)$, with $\omega = e^{2i\pi/3}$. The fourth column consists of two subcolumns which display for the lattice $\Lambda(SU(3))$, the fixed points and shift vectors, f_{n_3}, u_{n_3}, for the elements Θ^1 and $\Theta^{2,4}$, and the range of n_3. The fifth column consists of three subcolumns which display for the lattice $\Lambda(SO(4))$, the fixed points and shift vectors, $f_{n_2,n_2'}$, $u_{n_2,n_2'}$, for the elements Θ^1 and Θ^3, and the range of n_2, n_2'. The last column displays the modes multiplicities $\mathcal{D}$ assigned in the various twisted sectors T_g, in the case $W_2' = 0$ with unresolved quantum number $n_2' = 0$, 1 for the orbifold group action on T_1^2. The entries for $\mathcal{D}$ correspond to the entries for the eigenvectors, $|\gamma(g)>$, of eigenvalues $\gamma(g)$.

$\begin{array}{c}Z_{6-II}\\Z_2\times Z_3\end{array}$	$\mathcal{N}_{fp}^I$	$T_1^2 = \Lambda(G_2)$	$|\gamma(g)>$	$\gamma(g)$	$T_2^2 = \Lambda(SU(3))$	n_3	$T_3^2 = \Lambda(SO(4))$	n_2 n_2'	$\mathcal{D}$
$\begin{array}{c}T_1+T_5\\T_{1,2}+T_{1,1}\end{array}$	$1,3,4$	$\begin{array}{c}f_\alpha=0\\u_\alpha=0\end{array}$	1	1	$\begin{array}{c}f_{n_3}=\begin{bmatrix}0,\ \frac{2}{3}e_3+\frac{1}{3}e_4\\ \frac{1}{3}e_3+\frac{2}{3}e_4\end{bmatrix}\\u_{n_3}=0,e_3,e_3+e_4\end{array}$	$0,1,2$	$\begin{array}{c}f_{n_2,n_2'}=\frac{n_2}{2}e_5+\frac{n_2'}{2}e_6\\u_{n_2,n_2'}=-n_2e_5-n_2'e_6\end{array}$	$0,1$ $0,1$	2
$\begin{array}{c}T_2+T_4\\T_{0,1}+T_{0,2}\end{array}$	$3,3,1$	$\begin{array}{c}f_a=\frac{a}{3}e_1,\ u_a=-ae_2\\ [a=0,1,2]\end{array}$	$\begin{pmatrix}f_0,\frac{1}{\sqrt{2}}(f_1+f_2)\\ \frac{1}{\sqrt{2}}(f_1-f_2)\end{pmatrix}$	$\begin{pmatrix}1\\-1\end{pmatrix}$	$f_{n_3},\ u_{n_3}$	$0,1,2$	$\begin{array}{c}f_\alpha=0\\u_\alpha=0\end{array}$	0 0	$\begin{pmatrix}2\\1\end{pmatrix}$
$\begin{array}{c}T_3\\T_{1,0}\end{array}$	$4,1,4$	$\begin{array}{c}f_{ab}=\frac{1}{2}(ae_1+be_2)\\u_{ab}=ae_1+be_2\\ [a,b=0,1]\end{array}$	$\begin{pmatrix}f_{00},\frac{f_{01}+f_{10}+f_{11}}{\sqrt{3}}\\ \frac{f_{01}+\omega f_{10}+\omega^2 f_{11}}{\sqrt{3}}\\ \frac{f_{10}+\omega^2 f_{01}+\omega f_{11}}{\sqrt{3}}\end{pmatrix}$	$\begin{pmatrix}1\\\omega^2\\\omega\end{pmatrix}$	$\begin{array}{c}f_\alpha=0\\u_\alpha=0\end{array}$	0	$\begin{array}{c}f_{n_2,n_2'}\\u_{n_2,n_2'}\end{array}$	$0,1$ $0,1$	$\begin{pmatrix}4\\2\\2\end{pmatrix}$

vectors u_f and sublattices $\Lambda_{\Theta^g}^I = (1 - \Theta^g)\Lambda(T_I^2)$ of the twisted sectors g are defined as follows for the three 2-d tori, T_I^2, with G_2, $SU(3)$, $SO(4)$ group weight lattices:

$$\Lambda(G_2):\ (1 - \Theta^{2,4})f_a = u_a = -ae_2,\ f_a = \frac{a}{3}e_1,\ [a = 0,1,2];\ (1 - \Theta^3)f_{a,b} = u_{a,b} = \frac{a}{2}e_1 + \frac{b}{2}e_2,\ [a,b = (0,1)];$$

$$\Lambda_\Theta^1 \sim (e_1, e_2),\ \ \Lambda_{\Theta^{2,4}}^1 \sim (e_1, 3e_2),\ \ \Lambda_{\Theta^3}^1 \sim (2e_1, 2e_2).$$

$$\Lambda(SU(3)):\ (1 - \Theta^g)f_{n_3} = u_{n_3} = [e_3, e_3 + e_4] = n_3e_3,\ f_{n_3} = [0, \frac{2e_3 + e_4}{3},\ \frac{e_3 + 2e_4}{3}],\ [n_3 = 0,1,2];$$

$$\Lambda_{\Theta^g}^2 \sim (e_3 - e_4, 3e_3),\ [g = 1,2,4].$$

$$\Lambda(SO(4)):\ (1 - \Theta^g)f_{n_2,n_2'} = u_{n_2,n_2'} = -n_2e_5 - n_2'e_6,\ f_{n_2,n_2'} = \frac{1}{2}(n_2e_5 + n_2'e_6),\ [n_2,\ n_2' = 0,1];$$

$$\Lambda_{\Theta^g}^3 \sim (2e_5, 2e_6),\ [g = 1,3]. \tag{B.2}$$

The orbifold fundamental group allows for three discrete Wilson lines: W_2, W_2' of order $N_2 = N_2' = 2$, around two dual one-cycles of T_3^2, and W_3 of order $N_3 = 3$ around the e_3 one-cycle of torus T_2^2. Upon turning on the Wilson lines, the T_g twisted sectors split into twisted subsectors, $(g, \gamma, n_3, n_2, n_2')$, labelled by the discrete parameters, $n_3 = 0,1,2$ and $n_2 = 0,1$, $n_2' = 0,1$, for the lattices $\Lambda(T_2^2) \sim \Lambda(SU(3))$ and $\Lambda(T_3^2) \sim \Lambda(SO(4))$. The complex phase parameters, $\gamma(g)$, describing the action of Θ^g on the fixed points of the lattice $\Lambda(T_1^2) \sim \Lambda(G_2)$ are needed to specify the degeneracy of string modes in the sectors $g = 2, 4$ and $g = 3$. b The string states are described by the $SO(8)$ group weight vectors, r^a, $[a = 1, \cdots, 4]$ for the right-moving fermion fields; the $E_8 \times E_8$ group weight vectors, P^I, $[I = 1, \cdots, 16]$ for the left-moving 16 coordinate fields; and the oscillator numbers, N_i^R, $N_i^L \in Z$, $[i = I, \bar{I};\ I,\ \bar{I} = 1,2,3]$ for the 3 right- and left-moving complex coordinate fields X^I of T^6. The string squared mass spectrum is evaluated in terms of these quantum numbers by means of the formula

$$\frac{M_R^2}{8m_s^2} = \sum_I (N_I^R \omega_I^{(g)} + N_{\bar{I}}^R \omega_{\bar{I}}^{(g)}) + \frac{1}{2} \sum_{a=1}^4 (r^a + gv^a)^2 - E_0^{(g)} - \frac{1}{2},$$

$$\frac{M_L^2}{8m_s^2} = \sum_I (N_I^L \omega_I^{(g)} + N_{\bar{I}}^L \omega_{\bar{I}}^{(g)}) + \frac{1}{2} \sum_{I=1}^{16} (P^I + X_{g,n_f}^I)^2 - E_0^{(g)} - 1,$$

$$[E_0^{(g)} = \frac{1}{2} \sum_I |g\hat{v}_I|(1 - |g\hat{v}_I|),\ g\hat{v}_I = gv_I \bmod 1 = gv_I - [gv_I],$$

TABLE VII: The $SO(8)$ group weight vectors, $r_v^{l,r}$ and $r_s^{l,r}$, assigned to the massless right moving modes, $M_R^2 = 0$, with left and right chiralities $(l,\ r)$. The vector and spinor representations, $(v,\ s)$ differ by the weight vectors $\rho_{l,r}$ assigned to the supercharge generators, $r_s^{l,r} = r_v^{l,r} + \rho_{l,r}$, $[\rho_{l,r} = (\pm\frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2}, \pm\frac{1}{2})]$. The line entries refer to the untwisted and twisted sectors, $g = 0, 1, \cdots, 4$. No massless mode solutions arise for right movers carrying oscillator exitations, $N_{I,\ \bar{I}}^R$.

g	r_v^l	$r_s^l = r_v + \rho_L$	g	r_v^r	$r_s^r = r_v + \rho_R$
0	$(\underline{100}, 0)$	$(+\frac{1}{2} - \frac{1}{2} - \frac{1}{2}, -\frac{1}{2})$	0	$(\underline{-100}, 0)$	$(-\frac{1}{2} + \frac{1}{2} + \frac{1}{2}, +\frac{1}{2})$
1	$(001, 0)$	$(-\frac{1}{2} - \frac{1}{2} + \frac{1}{2}, -\frac{1}{2})$	5	$(-1 - 22, 0)$	$(-\frac{1}{2} - \frac{3}{2} + \frac{5}{2}, +\frac{1}{2})$
2	$(001, 0)$	$(-\frac{1}{2} - \frac{1}{2} + \frac{1}{2}, -\frac{1}{2})$	2	$(-1 - 11, 0)$	$(-\frac{1}{2} - \frac{1}{2}\frac{3}{2}, +\frac{1}{2})$
3	$(0 - 12, 0)$	$(-\frac{1}{2} - \frac{3}{2} + \frac{3}{2}, -\frac{1}{2})$	3	$(-1 - 11, 0)$	$(-\frac{1}{2} - \frac{1}{2}\frac{3}{2}, +\frac{1}{2})$
4	$(0 - 12, 0)$	$(-\frac{1}{2} - \frac{3}{2} + \frac{3}{2}, -\frac{1}{2})$	4	$(-1 - 22, 0)$	$(-\frac{1}{2} - \frac{3}{2} + \frac{5}{2}, +\frac{1}{2})$

$$X_{g,n_f} = gV + n_{f,a}W_a = gV + n_3 W_3 + n_2 W_2 + n_2' W_2'] \tag{B.3}$$

(in units of m_s) where X_{g,n_f} denote the shift vectors in the $E_8 \times E_8$ group lattice depending on the orbifold and Wilson lines gauge embeddings, V, W_3, W_2, W_2'. The oscillator energies are defined by $\omega_I^{(g)} = g\hat{v}_I$ for $g\hat{v}_I > 0$ and $1 - |g\hat{v}_I|$ for $g\hat{v}_I \leq 0$, with $N_{I,\ \bar{I}}^{L,R} \in Z_+$. An equivalent definition is, $\omega_I^{(g)} = gv_I \bmod 1$, $\omega_{\bar{I}}^{(g)} = -gv_I \bmod 1$, for the determinations obeying, $0 < \omega_{I,\bar{I}}^{(g)} \leq 1$. In the Z_{6-II} orbifold, the string states generally occur in CPT conjugate pairs of opposite 4-d chirality for the sectors, g and $g' = 6 - g$, except for the twisted sectors T_2 and T_4 with $g = 2,\ 4$, which admit both left and right chirality modes each (because of the supersymmetry $\mathcal{N} = 2$), and the (self-conjugate) twisted sector T_3 with $g = 3$. The vector and spinor weight vectors $r_{v,s}^a$ of the $SO(8)$ symmetry group for right moving fermions are associated to boson and fermion superpartners of the massless chiral supermultiplets as displayed in Table VII.

The GSO projection on the orbifold singlet states is determined via the one-loop partition function by the condition, $P(g, n_f, \gamma, \phi) = 1$, where

$$P(g, n_f, \gamma, \phi) = \frac{1}{N} \sum_{h=0}^{N-1} \Delta^h(g, n_f, \gamma, \phi),$$
$$[\Delta^h(g, n_f, \gamma, \phi) = \gamma(g,h)\phi(g,h)e^{2i\pi[(P+X_{g,n_f})\cdot X_{h,n_f} - (r+gv)\cdot hv - \frac{1}{2}(X_{g,n_f}\cdot X_{h,n_f} - ghv^2)]}]. \tag{B.4}$$

The twists along the world sheet spatial and temporal directions are denoted by g and h and the terms Δ^h include the complex phases, $\gamma(g,h) = \gamma^h(g)$, eigenvalues of the orbifold twist action on the fixed points in the 2-d torus lattice $\Lambda(T_1^2) = \Lambda(G_2)$, and the complex phases from oscillator excitations, $\phi(g,h) = \phi^h(g)$,

$$\phi(g) = e^{2i\pi \sum_{i=I,\bar{I}}(N_i^L - N_i^R)\hat{\phi}_i}, \quad [\hat{\phi}_I = v_I \mathrm{sgn}(g\hat{v}_I),\ \hat{\phi}_{\bar{I}} = -v_I \mathrm{sgn}(g\hat{v}_I),\ I, \bar{I} = 1, \cdots, 3]. \tag{B.5}$$

The level matching for right and left movers entails the conditions, $N(X_{g,n_f}^2 - (gv)^2)) = 0 \bmod 2$, with additional conditions involving the scalar products of Wilson lines, W_3, W_2, W_2'. For the twisted sectors, $T_{2,4}$ and T_3, the projections for the Z_3 and Z_2 suborbifolds can be implemented by summing over the time twists in the subsectors $(h = 3,\ m_2,\ m_2')$ and $(h = 2,\ m_3)$ as follows

- $g = (2,\ 4)$, $h = 3$: $P(g, n_3, \gamma, \phi) = \gamma^h(g)\phi^h(g)\frac{1}{4} \sum_{m_2, m_2'=0,1} e^{2i\pi[(P+\frac{1}{2}(gV+n_3 W_3))\cdot(hV+m_2 W_2+m_2' W_2') - (r+\frac{1}{2}gv)\cdot hv]}$,

- $g = 3$, $h = 2$: $P(g = 3, n_2, n_2', \gamma, \phi) = \gamma^h(g)\phi^h(g)\frac{1}{3} \sum_{m_3=0,1,2} e^{2i\pi[(P+\frac{1}{2}(gV+n_2 W_2+n_2' W_2'))\cdot(hV+m_3 W_3) - (r+\frac{1}{2}gv)\cdot hv]}$

$$\tag{B.6}$$

Except for the singly twisted sector, T_1, where the conditions in Eq. (B.4) hold with fixed values of n_3, n_2, the projections in the other sectors can be concisely stated in terms of simple sets of conditions on the weight vectors P^I. For the untwisted sector, these conditions are: $P \cdot V \in Z$, $P \cdot W_2 \in Z$, $P \cdot W_2' \in Z$, $P \cdot W_3 \in Z$. For the twisted sectors, $T_{2,4}$ and T_3, the projections are more easily implemented by imposing the conditions

- $g = (2,\ 4)$, $h = 3$: $\left(\begin{array}{c} \Delta^h = (\gamma\phi)^h e^{2i\pi h[(P+\frac{1}{2}(gV+n_3 W_3))\cdot(V+n_3 W_3) - (r+\frac{1}{2}gv)\cdot v]} = 1 \\ h\tilde{P} \cdot W_2 \in Z,\ h\tilde{P} \cdot W_2' \in Z,\ [\tilde{P} = P + X_{g,n_3,0,0}] \end{array} \right).$

- $g = 3,\ h = 2:$ $\left(\begin{array}{c} \Delta^h = (\gamma\phi)^h e^{2i\pi h[(P+\frac{1}{2}(gV+n_2 W_2+n'_2 W'_2))\cdot(V+n_2 W_2+n'_2 W'_2)-(r+\frac{1}{2}gv)\cdot v]} = 1 \\ h\hat{P}\cdot W_3 \in Z,\ [\hat{P} = P + X_{g,0,n_2,n'_2}] \end{array} \right).$ $\qquad$ (B.7)

This is the prescription for the GSO orbifold phase that we have used to obtain the string spectra of models $A1$, $A1$, B. Use of the world sheet modular invariance (level matching conditions) allows replacing the conditions on the scalar products in Eqs. (B.7) by the simpler ones [10]: $P\cdot W_2 \in Z$, $P\cdot W'_2 \in Z$ for $g = (2,\ 4)$ and $P\cdot W_3 \in Z$ for $g = 3$. The following alternative projections are also proposed in [15]

- $g = (2,\ 4):$ $\left(\begin{array}{c} 3[(N_i^L - N_i^R)\hat{v}_i + q_\gamma(g) - (r+gv)\cdot v + (P+gV+n_3 W_3)\cdot V] \in Z \\ (P+gV+n_3 W_3)\cdot \binom{W_2}{W'_2} \in Z \end{array} \right),$

- $g = 3:$ $\left(\begin{array}{c} 2[(N_i^L - N_i^R)\hat{v}_i + q_\gamma(g) - (r+gv)\cdot v + (P+gV+n_2 W_2+n'_2 W'_2)\cdot V] \in Z \\ (P+gV+n_2 W_2+n'_2 W'_2)\cdot W_3 \in Z \end{array} \right),$ $\qquad$ (B.8)

where $\gamma(g) = e^{2i\pi q_\gamma(g)}$. It is important to remark at this point that the orbifold projection for sectors with fixed T^2 subtori boils down to the invariance under the world sheet modular group. The modular invariance is vital if the mixed gauge and gravitational anomalies in the various Abelian gauge factors $U(1)_a$ are to satisfy the universality relations [81, 84]

$$\frac{1}{24k_a^{1/2}}Tr(Q_a) = \frac{1}{3k_a^{3/2}}Tr(Q_a^3) = \frac{1}{2k_a^{1/2}}Tr(Q_a c_b(R)) = \frac{1}{k_a^{1/2}k_b}Tr(Q_a Q_b^2) = 8\pi^2 \delta_{GS}^a, \qquad (B.9)$$

which are necessary for the Green-Schwarz type anomaly cancellation. mechanism to work. When the orbifold space group action by $h = (\Theta^h, u_{h,f})$ along the string time direction does not commute with that of the constructing orbifold element along the string space direction, $g = (\Theta^g, u_{g,f})$, the physical states must be constructed by summing over elements of the orbit, $h^n g h^{-n}$, and including suitable complex phase factors to ensure the $SL(2,Z)$ modular group invariance [17, 20, 85]. This situation indeed occurs for the sectors, $g = 2, 4$ and $g = 3$, where the fixed points in T_1^2 with lattice $\Lambda(G_2)$ are reshuffled by the action of h. The physical string modes decompose then into eigenvectors of Θ^g, $[g = 2, 3, 4]$ with eigenvalues $\gamma(g) = \pm 1$ in the $(g = 2,\ 4)$ sectors T_2, T_4 and $\gamma(g) = 1, \omega, \omega^2$, $[\omega = e^{2i\pi/3}]$ in the $(g = 3)$ sector T_3. The degeneracy of eigenstates is $\mathcal{D} = 2$ in T_1; $\mathcal{D} = (1, 2)$ for $\gamma = (1, -1)$ in T_2, T_4, and $\mathcal{D} = (4, 2, 2)$ for $\gamma = (1, \omega^2, \omega)$ in T_3, as displayed in Table VI.

For non-commuting twists, $[h, g] \neq 0$, the complex phase $\gamma(g, h)$ is not determined in a unique way by the GSO projection. Since $\gamma(g, h)$ fixes the degeneracies of string modes, its choice has clearly an incidence on the $U(1)_a$ groups anomalies and hence on the modular invariance. The choice of $\gamma(g, h)$ can be set uniquely by requiring the mixed gauge and gravitational anomalies to satisfy the anomalies universality relations in Eq. (B.9). Given a massless string state, specified by the quantum numbers, P^I, R^I, N_i^L, then the freedom on $\gamma(g, h)$ is taken into account by redefining, $\gamma(g, h) \to \chi(h)\gamma(g, h)$, in terms of the GSO modified condition, $\Delta^h \chi^h = 1$, so that the mode multiplicities determined by $\gamma(g, h)$ give anomaly coefficients satisfying the requisite universality relations. Although the natural choice, $\chi = 1$, applies for the large majority of modes, there do occur cases where a non-trivial χ in Δ^h is needed to satisfy the universality relations. The factors $\chi(h)$ depend on the prescription used for the modified gamma phases. With our GSO projection prescription in Eqs.(B.7), one exception occurs for three modes of sector T_3 in model $A2$ and another one for two modes of sectors T_2 and T_4 in model B.

Finally, we quote the string selection rules on the n-point world sheet correlators of vertex operators, $< \prod_{l=1}^n V_l(z_l, \bar{z}_l) >$. Applied to the superpotential couplings, the conditions set by the gauge symmetries, the H-momentum conservation, and the orbifold point and space symmetry groups read:

- $\sum_{l=1}^n [P^I(l) + X_{g,n_f}^I(l)] \in \Lambda(E_8 \times E_8)$, $[X_{g,f} = gV + n_{g,f}^a W_a]$.

- $[\sum_{l=1}^n R_I(l) - 1] = 0 \mod (\frac{1}{|v_I|}) = 0 \mod (6, 3, 2)$, $[R^I(l) = r^I(l) + g_l v^I(l) - N_I^L(l) + N_{\bar{I}}^L(l)]$.

- $\sum_{l=1}^n \Theta_I^{g_l} = 0 \mod 1 \implies \sum_{l=1}^n g_l = 0 \mod N$.

- $\sum_{l=1}^n (1 - \Theta_I^{g_l})(f_l - \Lambda_I) = 0 \mod N \implies \sum_{l=1}^n n_3(l) = 0 \mod 3$, $\sum_{l=1}^n (n_2(l), n'_2(l)) = (0, 0) \mod 2$. $\qquad$ (B.10)

Some of the above selection rules can be formulated as ordinary [11, 86] or R type [87] discrete symmetries. We have omitted from the list in Eq.(B.10) the constraints on couplings from the space orbifold group action on the T_1^2 torus with lattice $\Lambda(G_2)$. These consist of the following two selection rules, stated around Eq. (C.9) of [10]:

- Rule I on the gamma phases of physical modes: $\prod_{l=1}^{n} \gamma_l = 1$. Using the definition, $\gamma_l \equiv \gamma(g_l) = e^{2i\pi q_{\gamma_l}}$, this can also be written as, $\sum_{l=1}^{n} q_{\gamma_l} = 0 \mod 1$.

- Rule II for couplings involving the twisted sector modes $T_{2,4}$ only, or twisted sector modes T_3 only, times untwisted sector states. The version of this rule derived in [13] states that the column vector of gamma phases for fields in n-point couplings $T_{2,4}^{p_1} T_0^{p_2}$ or $T_3^{p_1} T_0^{p_2}$, with $p_1 \neq 0$, $n > p_2 > 0$ must obey the condition, $\vec{q}_{\gamma_l} \notin [(p, 0, \cdots, 0) + \text{perms}]$, $[p \neq 0]$. Thus, the allowed couplings are those containing at least two physical modes with non-trivial gamma phases, $\gamma_l \neq 1$. The version of this rule derived in [10] uses instead the conjugacy classes of the lattice cosets, $\Lambda/\Lambda_{\Theta^g}$, associated to the fixed points $f_{a_l}^l = \frac{a_l}{3} e_1$, $[a_l = 0, 1, 2]$ in sectors $g = 2, 4$ and $f_{a_l, b_l}^l = \frac{a_l}{2} e_1 + \frac{b_l}{2} e_2$, $[a_l = 0, 1; \, b_l = 0, 1]$ in sector $g = 3$, in the notational conventions of Table VI and Eq. (B.2). In terms of the fixed point content of the physical states (with fixed γ_l eigenvalues), the orbifold space group constraints are described for the couplings $T_{2,4}^{p_1} T_0^{p_2}$ by the Z_3 group selection rule, $\sum_{l=1}^{n} a_l = 0 \mod 3$, and for the couplings $T_3^{p_1} T_0^{p_2}$ by the $Z_2 \times Z_2$ group selection rule, $\sum_{l=1}^{n} a_l = 0 \mod 2$, $\sum_{l=1}^{n} b_l = 0 \mod 2$.

Convincing arguments are given in [13, 20, 85] that Rule I is not a genuine selection rule, since it is found to be automatically satisfied by physical states obeying the GSO orbifold projection, $P(g, n_f, \gamma, \phi) = 1$, once the selection rules on $\Theta_I^{g_l}$, $P^I(l)$, $R^I(l)$ are imposed. In a consistent string model complying with the orbifold GSO projection, and hence with the world sheet modular group invariance, Rule I is redundant. However, the redundancy was proved with the specific prescription for the orbifold projection quoted in Eq.(B.8). Since our prescription in Eq.(B.7) is quite close but not strictly identical to that used in [13, 20], there is no guarantee that Rule I is automatically implied by other rules in our case too. That a selection rule is superfluous in one projection prescription and not in another appears odd at first sight but cannot be excluded. It certainly is a logical possibility if several consistent string models arise from the same data set of shift vectors, V, W_2, W_3. To settle this issue on practical grounds, we have applied our search procedure for models $A1$, B with the rule $\prod_{l=1}^{n} \gamma_l = 1$ included, and found that this did forbid certain superpotential couplings allowed by the rules in Eq.(B.10). However, the excluded couplings represent a very tiny fraction of the large set of allowed couplings, which cause insignificant changes on the effective couplings. We have also tested the Rule II above and found that this had a negligible incidence on the allowed couplings.

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